Exam 1 review sheet

Part 1: Wednesday March 18. 11:30am-12:30.[1] Closed book, closed note, no calculator.

Part 2: 2-hour block on Wed or Thurs. Open book.

Present *clear* and *complete* answers. Show your work in some coherent fashion. Unjustified answers will earn no points. Well-reasoned answers can receive partial credit. Start all solutions with definitions, facts, or commonly used equations. Include calculations, explanations, and diagrams when appropriate.

Ch 2, 3 and 5

1. Given the forces on a particle (such as gravity, a retarding force, etc), determine the
   (a) differential equation that governs the particle's motion
   (b) equations of motion, $x(t)$ and $v(x)$
   (c) terminal velocity, if applicable

2. Determine the equations of motion specific to a given set of initial conditions. (That is, find the constants!) This is also applicable to oscillator problems.

3. Given a force,
   (a) determine if it is conservative or non-conservative. Justify your answer with a calculation and some text.
   (b) If it is conservative, find the potential energy associated with it.
   (c) Alternatively, given a potential energy, find the force associated with it.

4. Starting from the statement of conservation of mechanical energy, $E = T + U$, derive the following general expression used to determine the equation of motion for a particle of mass m

$$t - t_0 = \int_{x_0}^{x} \frac{\pm dx}{\sqrt{\frac{2}{m}(E - U(x)}}$$

5. Use conservation of energy to find a particle's equations of motion, $x(t), v(x)$.

[1] Make-up exams will only be arranged if you can provide (1) advance notice and (2) a reasonable and documentable excuse.

6. Given initial conditions and the potential energy $U(x)$, find a particle's
   (a) equilibrium point(s) and determine the stability
   (b) the turning points
   (c) and kinetic energy

7. Consider a particle of mass m subject to a few (given) forces. Determine whether this particle will undergo simple harmonic motion. Justify your answer with a calculation or by deriving a relevant equation.

   If it does undergo simple harmonic motion, determine its period.

8. Consider a particle undergoing simple harmonic motion. Staring from its equation of motion, $x(t) = A\sin(\omega_0 t - \delta)$, show using explicit calculations that its total energy is conserved and that $E = \frac{1}{2}kA^2$.

9. Consider a particle of mass m subject to a restoring force ($-kx$) and a frictional force ($= -bv$).
   (a) Starting from Newton's second law, derive the following general equation of motion

$$x(t) = e^{-\beta t}\left[A_1 e^{\sqrt{\beta^2-\omega_0^2}t} + A_2 e^{-\sqrt{\beta^2-\omega_0^2}t}\right]$$

   where $\omega_0^2 = k/m$ and $\beta = b/2m$.
   (b) Show that in the case of underdamping,

$$x(t) = e^{-\beta t}\left[B_1 \sin(\omega_1 t) + B_2 \cos(\omega_1 t)\right]$$

   What are ω_1, B_1, and B_2?
   (c) Show that in the case of overdamping,

$$x(t) = e^{-\beta t}\left[C_1 e^{\omega_2 t} + C_2 e^{\omega_2 t}\right]$$

   What are ω_2, C_1, and C_2?
   (d) When does critical damping occur? Is the $x(t)$ given in (a) still applicable? If it isn't, what is the appropriate $x(t)$?

10. Consider a particle of mass m subject to a restoring force ($-kx$) , a damping force ($-bv$) and a sinusoidal driving force ($F_0 \cos\omega t$). Derive the following differential equation that the governs the motion of this particle:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = A\cos\omega t$$

    What are β, ω_0 and A? Is $\omega = \omega_0$?

11. The sinusoidally driven oscillator is said to have transient and steady state solutions. Why are there two solutions? Does the oscillator choose one over the other? How are they different?

12. What is resonance? Does it occur for all harmonic oscillators? Explain.

13. Starting from the particular solution to the sinusoidally driven oscillator,

$$x(t) = \frac{A}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\omega^2\beta^2}} \cos(\omega t - \delta)$$

show that resonance frequency ω_R is $\sqrt{\omega_0^2 - 2\beta^2}$.

14. Express a given function as a Fourier series.

15. Given a set of point masses or a continuous distribution mass, determine the gravitational
    - (a) field at a particular point
    - (b) potential at a particular point
    - (c) force exerted on a point mass placed near the masses
    - (d) potential energy associated with a point mass placed near the masses.

16. Calculate the gravitational field from the gravitational potential.

17. Use Gauss's law to determine the gravitational field of a highly symmetric distribution of mass.

    Why don't we use Gauss's law to calculate the field for all mass distributions?

Equations provided for Part 1

binomial expansion

gradient and curl in spherical coordinates

trig series for $\sin x, \cos x$ (e.g. $\sin x = x - x^3/3! \ldots$)

Math equations you should know for Part 1 (provided at a cost)

sine, cosine and tangent dftns ($\sin\theta =$ opp/adj ...)
solution to the quadratic equations ($x = b^2 \pm \ldots$)
Euler's formula ($e^{i\theta} = \cos\theta + \ldots$)
the curl and divergence in rectangular coordinates
dot and cross products
relationship between angle, radius and arc ($s = r\theta$)
volumes, surface areas of spheres, cylinders