# Phys 105- Exam 1, part 1

1/ (a) To be conservative, $\vec{F}$ must be able to be written as $\vec{F} = -\vec{\nabla} U$.

This will be true for any $\vec{F}$ when $\vec{\nabla} \times \vec{F} = 0$ (see Stoke's theorem).

Check that $\vec{\nabla} \times \vec{F} = 0$ ...

$$\vec{\nabla} \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & 0 & 0 \end{vmatrix} = \frac{\partial F_x}{\partial z}\hat{j} - \frac{\partial F_x}{\partial y}\hat{k}$$

$$= 0 - 0 = 0 \checkmark$$

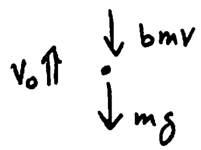
since $F_x$ only has x-dependence.

(b) $\vec{F} = -\vec{\nabla} U$

$$F_x = -\frac{\partial U}{\partial x} \rightsquigarrow U = -\int F_x \, dx = -\int (x - x^3)\, dx$$

$$\boxed{U = -\frac{x^2}{2} + \frac{x^4}{4} + \text{const.}}$$

2/




So apply Newton's second law $\quad \sum F = m\ddot{y}$

$$-mg - bmv = m\ddot{y}$$

rewrite in terms of velocity, simplify

$$-(g + bv) = \frac{dv}{dt}$$

$$-\int dt = \int \frac{dv}{g + bv}$$

$$-t + C = \frac{1}{b} \ln (g + bv)$$

apply initial condition $v = v_0$ at $t = 0$

$$0 + C = \frac{1}{b} \ln (g + bv_0)$$

$$-t + \frac{1}{b} \ln (g + bv_0) = \frac{1}{b} \ln (g + bv)$$

$$g + bv = e^{-bt + \ln (g + bv_0)}$$

$$v = \frac{(g + bv_0)}{b} e^{-bt} - \frac{g}{b}$$

3/ to be at equilibrium, $\sum \vec{F} = 0$. If there's only one conservative
(a) force, this means $\vec{F} = -\vec{\nabla} U = 0$

so check first derivative
$$\left.\frac{dU}{dx}\right|_{x=\text{critical}} = U_0 \left(1 - e^{-(x-x_0)/\delta}\right)(-1)\left(-\frac{1}{\delta}\right) e^{-(\;)/\delta} = 0$$

therefore
$$\left(1 - \underbrace{e^{-(x-x_0)/\delta}}_{\text{must } = 1}\right) = 0$$

so exponent must $= 0$

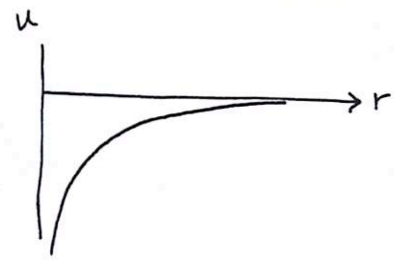
$$\frac{(x-x_0)}{\delta} = 0$$

$$x = x_0$$

(b) to check stability, calc $\left.\dfrac{d^2U}{dx^2}\right|_{x=x_0}$

to be stable $\left.\dfrac{d^2U}{dx^2}\right|_{x=x_0} > 0$

4/ to escape the potential ...




must have just enough KE to get to $r \to \infty$

$$E_i = E_f$$

$$\frac{1}{2} m v_{esc}^2 - \frac{GMm}{R_E} = \cancel{\frac{GMm}{r}} + \cancel{\frac{1}{2} m v^2}^{\,0}$$

0 since $r \to \infty$

$$v_{esc} = \sqrt{\frac{2GM}{R_E}}$$

5/(a) Apply Newton's law.. $\sum F = m\ddot{y}$

$$-3b m\dot{y} - \frac{17}{2}b^2 m y = m\ddot{y}$$

simplify $\quad \ddot{y} + 3b\dot{y} + \frac{17}{2}b^2 y = 0$

let $y = e^{rt}$ and determine app. r's

$$r^2 e^{rt} + 3bre^{rt} + \frac{17}{2}b^2 e^{rt} = 0$$

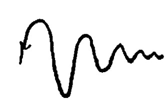
$$r^2 + 3br + \frac{17}{2}b^2 = 0$$

$$r = -\frac{3}{2}b \pm \frac{b}{2}\sqrt{9-34}$$

$$= -\frac{3}{2}b \pm i\frac{5b}{2}$$

so $\quad y(t) = e^{-\frac{3}{2}bt}\left[A e^{+i\frac{5}{2}bt} + B e^{-i\frac{5}{2}bt}\right]$

(b) this is an underdamped oscillator.
Note that this would lead to a decaying oscillation as indicated by the $e^{i\omega t}$ which could be easily turned to sines & cosines



6/(a) $\delta$ represents the shift (or lag) of the response with respect to F(t). That is, the maximum in F(t) doesn't necessarily have to occur when the oscillator is at a maximum displacement!

(b) the resonance frequency is when the amplitude is a maximum...

$$A = \frac{F_0/m}{\left[(\omega_0^2-\omega^2)^2 + 4\omega^2\beta^2\right]^{1/2}}$$

$$\left.\frac{dA}{d\omega}\right|_{\omega=\omega_R} = 0 = \frac{F_0}{m}\left[(\omega_0^2-\omega_R^2)^2 + 4\omega_R^2\beta^2\right]^{-1/2}\left[2(\omega_0^2-\omega_R^2)(-2\omega_R) + 8\omega_R\beta^2\right]$$

so to be zero,
the numerator
must go to zero...

$$2(\omega_0^2 - \omega_R^2)(-2\omega_R) + 8\omega_R\beta^2 = 0$$

$$-4\omega\left[-\omega_0^2 + \omega_R^2 + 2\beta^2\right] = 0$$

or $$\omega_R^2 = \omega_0^2 - 2\beta^2$$