NAME:_____

Exam 3
May 3, 2019

Some direction:

No phones or other device that connects to the internet.

You may use a calculator, **plug in numerical answers** and keep three sig figs regardless of the correct number of sig figs in the problem.

Present clear and complete answers:

Explain your answers clearly but briefly. You want to aim for a level of solution that someone taking this class would be able to understand. A diagram and a few words may help.

Start calculations with first principles: things like definitions ($\vec{E} \equiv \frac{\vec{F}}{Q}$) or empirical laws (like Coulomb's Law or Newton's Laws) or conservation laws.

Check time:

The point values for each problem are shown next to the question number. Time yourself accordingly. The total value of the exam is 100 points. **Good luck!**

Some constants:

$$K = k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 Nm^2/C^2 \approx 9 \times 10^9 Nm^2/C^2$$

$$\epsilon_0 = 8.85 \times 10^{-12} C^2/N/m^2$$

$$\mu_0 = 4\pi \times 10^{-7} Tm/A$$

$$e = 1.6 \times 10^{-19} C$$

Helpful Equations:

$$E_i = E_f$$

$$E = K + U$$

Definitions	Point charges	Uniform $\vec{E}$
	$\lvert\vec{F}\rvert = k\frac{\lvert qQ\rvert}{r^2}$	$\vec{F} = Q\vec{E}$
$\vec{E} \equiv \frac{\vec{F}}{Q}$	$\lvert\vec{E}\rvert = k\frac{\lvert q\rvert}{r^2}$	$\vec{E} = const$
$\Delta U \equiv -W$	$U = k\frac{qQ}{r}$	$\Delta U = QEs$
$V \equiv \frac{U}{Q}$	$V = k\frac{q}{r}$	$\Delta V = Es$

$$dq = \lambda d\ell$$

$$dq = \sigma da = \eta da$$

$$dq = \rho dV$$

$$\Phi = \int \vec{E} \cdot d\vec{A}$$

$$\int \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$C \equiv \frac{|Q|}{|V|}$$

$$C = \epsilon_0 \frac{A}{d}$$

$$W = \frac{1}{2}CV^2$$

$$I = \frac{dQ}{dt} = \frac{\Delta Q}{\Delta t}$$

$$V = IR$$

$$R_{eq} = R_1 + R_2 + ...$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ...$$

$$q(t) = Q_{max}(1 - e^{-\frac{t}{RC}})$$

$$q(t) = Q_{max}e^{-\frac{t}{RC}}$$

$$P = IV$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ...$$

$$C_{eq} = C_1 + C_2 + \ldots$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{\ell} \times \hat{\boldsymbol{r}}}{\boldsymbol{r}^2}$$

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{enc}$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$I = |\vec{I}| = \int \vec{J} \cdot d\vec{A}$$

$$\vec{F} = \int I d\vec{\ell} \times \vec{B}$$

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B dA \cos\theta$$

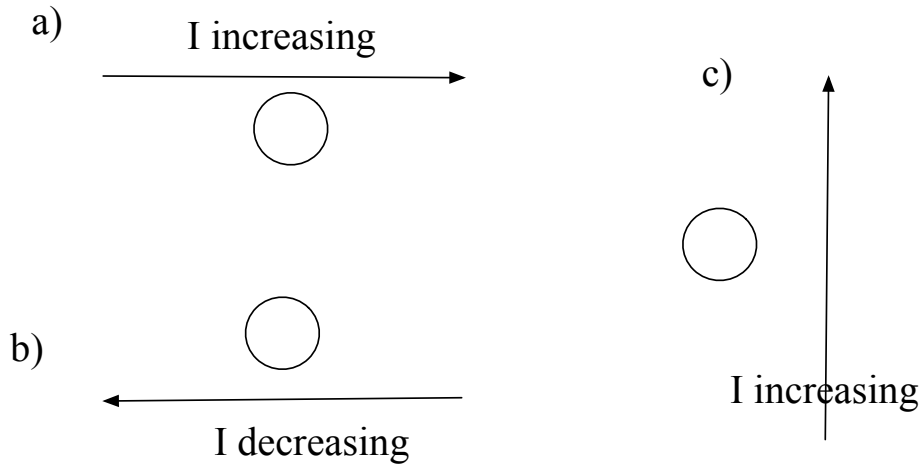
$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

1. (5 points) Mass causes gravitational fields. Charge causes electric fields. What causes magnetic fields?

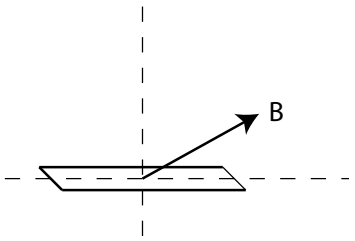
2. (5 points) A long straight wire carries a current of 30A due North. What is the magnetic field (magnitude and direction) 25cm due East of the wire? (Be very careful about how you describe the direction of your answer.)

3. (10 pts) An electron traveling with velocity 3 m/s $\hat{z}$ enters a region with both an electric field and magnetic field. The magnetic field, $\vec{B} = .015\text{T}\ \hat{x}$. Find the magnitude and direction of $\vec{E}$ such that the electron continues at the same velocity.

4. (10 points) The following three figures represent a long straight wire carrying current I next to a wire loop. I increases or decreases as shown. What direction is the induced current in each loop?



5. (10 pts) A wire loop in the shape of a rectangle has dimensions 30cm by 40cm. The magnetic field in the region is 40mT at an angle of 30° up with respect to the plane of the loop as shown below.



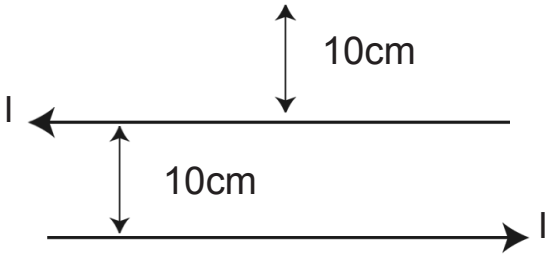
(a) What is $\vec{A}$ for the rectangular loop. (There are two possible correct answers. Either (or both) is fine.)

(b) What is the magnitude of the magnetic flux through the loop?

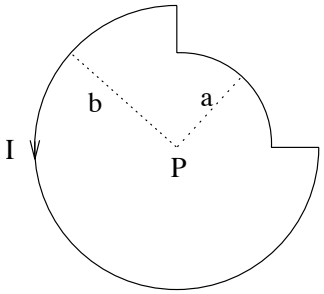
6. (15 pts) Two very long straight wires separated by a distance of 10cm each carry a current of 4A, but in opposite directions as shown.

   (a) What is the magnetic field (magnitude and direction) 10cm above the top wire?

   (b) Now a third very long straight wire, with current of 3A to the right, is placed 10cm above the first two wires. What is the magnitude and direction of the force per unit length on the third wire?



7. (15 pts) A current carrying wire is bent into the rather odd shape shown below. Each curve is a piece of a circle, for example, a quarter of a circle of radius a and 3/4 of a circle of radius b. The straight pieces are along radial lines from the center. Starting from the Biot-Savart Law, find the magnetic field at point P, which is at the "center" of the figure.

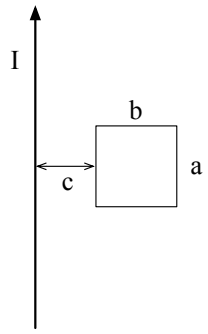


8. (15 pts) Find the magnitude of the magnetic field a distance r from the center of a very long wire in the shape of a cylinder of radius a carrying a total current I to the right (along the axis of the cylinder). The current is distributed uniformly throughout the cross-sectional area of the cylinder. (Even if you know the answer, show all the work to get it.)

   (a) Find the magnetic field outside the cylinder ($r > a$)

   (b) Find the magnetic field inside the cylinder ($r < a$)

9. (15 pts) A rectangular loop of sides a and b is a distance c from a long straight wire carryng current I as shown.



   (a) Show that the flux through the loop is

$$\frac{\mu_0 I a}{2\pi} \ln\left(\frac{c+b}{c}\right)$$

   (b) If the current in the long straight wire is $I = I_0 \cos\omega t$ what is the induced emf in the loop?