2/14 In Class – Vector Review, Coulomb's Law, and maybe E fields

((Most groups made it through number 5 in class. We will begin with number 6 next time.))

Some guidelines for in class group work

Let's work on the white boards today. Let's start with six groups, which means about four people per group. Two groups at front board, two at side, two groups in back.

Introduce yourselves! Please introduce yourselves to each other. Please write your names on the white board. I will come around and introduce myself and take pictures so I can learn your names.

Work in groups – the entire group should be working on the same problem. (No divide and conquer strategy.) This is a bit like shared inquiry you might know about from Seminar. We all learn from each other, and we all learn best by explaining our process to someone else. Also, please go in order. Sometimes, these are tutorials, and build on the previous problem(s). Don't erase (if at all possible) until I have checked it off.

If you get stuck and I am with another group, please feel free to introduce yourselves to another group and ask them for help. Or you can look around the room and see if another group has it on the board.

Write enough on the board that someone else could follow your solution. In fact, I'll take pictures of the answers on the boards, and they will become our solution set.

Vector review and maybe new notation

I'm a big fan of writing vectors using column notation. So, for example, a vector

$$\vec{v} = (3\hat{x} + 4\hat{y})\text{m/s} = (3\imath + 4\jmath)m/s = \begin{bmatrix} 3 \\ 4 \end{bmatrix} m/s$$

where the top component of the column is the x-component and the bottom is the y-component.

The main advantage of this is: when you plug these into a vector equation, you are less likely to forget or mix up components. In this notation,

$$\vec{g} = \begin{bmatrix} 0 \\ -g \end{bmatrix} = \begin{bmatrix} 0 \\ -9.8 \end{bmatrix} m/s^2$$

So, for example, the kinematic equations look like:

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} v_{0x}t \\ y_{0y}t \end{bmatrix} + \begin{bmatrix} \frac{1}{2}a_x t^2 \\ \frac{1}{2}a_y t^2 \end{bmatrix}$$

In the last equation you can see the power of this method. You have both the entire thing, and if you read just the top row (which is an equation in itself) you get the 1D kinematic equation for the x-component, and the bottom row is y.

The same thing works for adding forces:

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

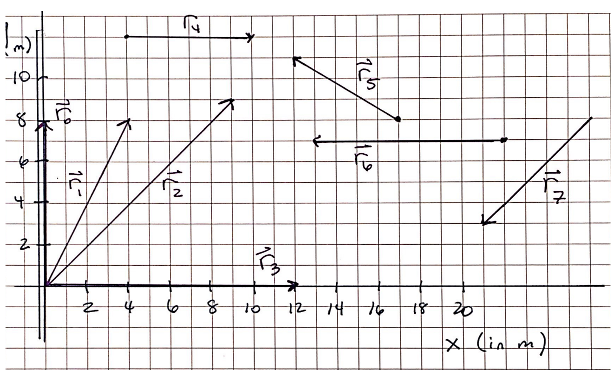
becomes:

$$\begin{bmatrix} F_{net,x} \\ F_{net,y} \end{bmatrix} = \begin{bmatrix} F_{1x} \\ F_{1y} \end{bmatrix} + \begin{bmatrix} F_{2x} \\ F_{2y} \end{bmatrix} + \begin{bmatrix} F_{3x} \\ F_{3y} \end{bmatrix}$$

where again, the top row is the sum of the x-components, and the bottom row is the sum of the y.

Let's do some practice with this notation. And some vector review!

1. Using the figure shown below, write vectors $\vec{r}_0$ through $\vec{r}_7$ as column vectors.



2. Using the results of the previous problem, evaluate the following: (please leave your answers as column vectors!)

   (a) $\vec{d}_1 = \vec{r}_0 + \vec{r}_3$

   (b) $\vec{d}_2 = \vec{r}_0 + \vec{r}_3 + \vec{r}_5$

   (c) $\vec{d}_3 = \vec{r}_2 + \vec{r}_5$

   (d) $\vec{d}_4 = \vec{r}_2 - \vec{r}_5$

   (e) $\vec{d}_5 = 3\vec{r}_0 + 2\vec{r}_5$

   (f) $\vec{d}_6 = 2\vec{r}_5 + \vec{r}_7$

   (g) $\vec{d}_7 = 2a\vec{r}_5 + a\vec{r}_7$ where a is a scalar. You will have an answer in terms of a.

3. A vector has a magnitude of 15m, and it points in the direction 30° up from the positive x-axis. Write this as a column vector.

4. A force is:

$$\vec{F} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} N$$

Find the magnitude and direction of this force.

Quick summary of Coulomb's Law

Coulomb's Law gives the force between two point particles that have charges of q_1 and q_2. The separation between the two charges is r. The magnitude of the force is given by:

$$F = K\frac{q_1 q_2}{r^2}$$

and the direction of the force is along the line between the two particles and away for like charges and toward for opposite charges.

If there are more than two charges, you calculate the force for each pair that includes the one you're interested in, then add the pairs of forces. Remember that forces are vectors, so you have to worry about directions, and you may have to use components.

5. An electron is 0.53 Å(the Bohr radius) from a proton. Find the magnitude of the electric force on the electron. Recall that $e = 1.6 \times 10^{-19}$C.

6. A point particle with a charge of $Q_1 = 1$ nC is at the origin. A second point particle with charge $Q_2 = -2$nC is at $x = 30$cm, and a third point particle with a charge of $Q_3 = 3$nc is at $x = 60$cm. Find the electric force on Q_3 due to the other two charges by the following steps:

   (a) Draw a picture of the configuration.

   (b) Draw a separate free body diagram of Q_3. Let's use Knight's convention that $\vec{F}_{13} = \vec{F}_{1on3}$ means the force of object 1 on object 3.

   (c) Use Coulomb's Law to evaluate the magnitudes of $\vec{F}_{13}$ and $\vec{F}_{23}$ You can leave this in symbols for now.

   (d) Find the net force on Q_3.

7. A point particle with a charge of $Q_1 = 1$ nC is at the origin. A second point particle with charge $Q_2 = 2$nC is at $y = 30$cm, and a third point particle with a charge of $Q_3 = 3$nc is at $x = 60$cm. Find the electric force on Q_3 due to the other two charges by the following steps:

(a) Draw a picture of the configuration.

(b) Draw a separate free body diagram of Q_3. Let's use Knight's convention that $\vec{F}_{13} = \vec{F}_{1on3}$ means the force of object 1 on object 3.

(c) Use Coulomb's Law to evaluate the magnitudes of $\vec{F}_{13}$ and $\vec{F}_{23}$ You can leave this in symbols for now.

(d) Find the net force on Q_3. You may leave your answer in components, as a column vector.

NEW: The electric field

Physicists were long unhappy with the notion of action-at-a-distance forces. Meaning, how do the two charges in the Coulomb Force affect each other if they don't touch? The answer: each charge generates a field everywhere in space.

The definition of the electric field, $\vec{E}$ is:

$$\vec{E} \equiv \frac{\vec{F}}{Q}$$

For point charges, this means that one point charge, q, generates a field of magnitude

$$E = K\frac{q}{r^2}$$

and you find the direction of the field at any point of interest by pretending to put a very small, positive test charge at that point and using Coulomb's Law to find the direction.

8. The electric field is a vector. It has no special name for its units. What is one way to write the units of $\vec{E}$?

9. Find the electric field a distance of 3m above a point particle with a +2C charge.

10. Find the electric field a distance of 3m below a point particle with a +2C charge.

11. Find the electric field a distance of 6m to the right of a point particle with a +2C charge.

12. Find the electric field a distance of 6m to the right of a point particle with a −2C charge.

13. A point particle with a charge of $Q_1 = 1$ nC is at the origin. A second point particle with charge $Q_2 = -2$nC is at $x = 30$cm,

    (a) What is the electric field due to those two particles at $x = 60$cm?

    (b) What force would a charge of 3nc feel at that same point (60cm)? How does this compare to your answer to problem 6?