3/15 in Class–Tutorial and Practice
with Capacitance, Gauss's Law, and some definitions

Recall that capacitance, C is defined as

$$C \equiv \frac{|Q|}{|V|}$$

and has units of Farads.

Recall also that we derived the expression for the capacitance of a parallel plate capacitor where the plates have area A and are separated by a distance d.

$$C = \epsilon_0 \frac{A}{d}$$

1. (This problem was on the last handout also.) Two parallel plates in the shape of circles have a radius of 15cm and a separation of 0.5 cm. A potential difference of 12,000 V is applied across the plates.

   (a) What is the capacitance of this arrangement?

   (b) How much charge is on each plate?

   (c) Recall that the field is uniform for a parallel plate capacitor. What is the magnitude of electric field anywhere between the plates?

   (d) What is the direction of the field: in terms of charge? in terms of potential?

2. **Energy stored in a capacitor**
   Let's start with two neutral parallel plates. At first there is no ΔV between the plates because they are neutral. Now imagine moving one tiny fraction of the total charge, $+dq$ from the plate on the right to the plate on the left. This leaves a $-dq$ on the plate on the right. Now there is a charge difference between the plates, and there is a ΔV between the plates. You can continue in this way until you get the full $+Q$ on the left plate and $-Q$ on the right plate. The mathematical expression that matches this thought experiment is:

$$U = \int V dq$$

   (a) You can't integrate that until you write V as a function of q. Use the definition of capactance to rewrite V. Then integrate q from 0 to Q. What do you get?

(b) Substitute back in for Q to get an expression for U in terms of only C and V. This is the most commonly written expression for energy stored in a capacitor. (Be extra sure to check this one with me.)

3. For the capacitor in problem 1, how much energy can that capacitor store?

4. (Taken from a tutorial at CU Boulder) A real coaxial cable (found in physics labs) has an inner conducting solid cylinder of radius a (i.e. a wire!) and an outer conducting cylindrical shell (inner radius b). It is physically easy to set up a fixed, given, potential difference ΔV between the inner and outer conductors, keeping the entire coax neutral. This problem will walk you through a way to find the capacitance for this system.

   (a) Use Gauss's Law to find the electric field in the region between the cylinders. ($a < r < b$ where r is radially out from a central axis.)

   (b) Now use $\Delta V = -\int \vec{E} \cdot d\vec{r}$ to find ΔV between the plates.

   (c) Now use the definition of capacitance to find the capacitance per unit length (I think you'll see why you can't just find C alone).

5. (Taken from a tutorial at CU Boulder) This (cylindrical capacitor from the last problem) is also a physical model for axons, which are long cylindrical cells (basically coax cables!) carrying nerve impulses in your body and brain. Using part c) above, estimate the capacitance (in SI metric units, which are Farads) of your sciatic nerve.

   Assumptions - the sciatic nerve is the longest in your body, it has a diameter of roughly 1 micron, and a length of perhaps 1 m(!) Axons generally have a value of b which is very close to a (i.e. the gap is extremely tiny, b-a is about 1 nanometer) so you can simplify your expression using $ln(1 + \epsilon) \approx \epsilon$. Think carefully about what ϵ is here, though!