4/12 In Class Practice Problems – Biot Savart and Ampere's Laws

First, a bit of review:

Recall the Biot-Savart Law is:

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{\ell} \times \hat{\boldsymbol{\imath}}}{\imath^{\,2}}$$

In class, I derived the equation for the magnetic field a distance h above the center of a circular loop of wire of radius R carrying current, I.

1. We often want $\vec{B}$ at the center of the circle (where $h = 0$). Use the Biot-Savart Law to derive this from scratch. So, find $\vec{B}$ for a circular loop of radius R in the plane of the paper. The current I travels in the clockwise direction. (Don't just take my answer and plug in $h = 0$, though you can check your answer that way.)

A bit more review, and a new equation:

In class, I derived the equation for the magnetic field a distance r above the center of a straight wire of length 2ℓ carrying current, I. I found:

$$B = \frac{\mu_0 I}{2\pi r}\left[\frac{\ell}{\sqrt{\ell^2 + r^2}}\right]$$

If the current is traveling to the right, and we are above the wire, the direction of $\vec{B}$ is out of the board or page. If we are below the wire, it's in. The magnetic field wraps around the wire.

For $\ell >> r$, this becomes:

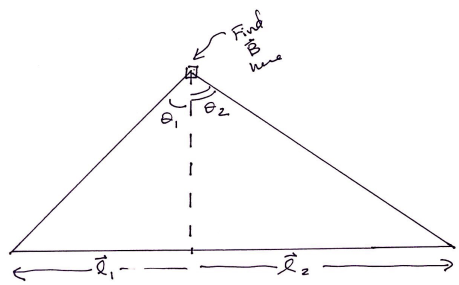
$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$

Recall that $\hat{\phi}$ is short hand notation for the direction that wraps around the wire. Sometimes we also call that circumferential.

New Stuff: If the wire is not symmetric, or if you are not above the center of the wire, here are two equations (that I derived from Biot-Savart), that can be used to find $\vec{B}$.

$$\vec{B} = \frac{\mu_0 I}{4\pi r}\left[\frac{\ell_2}{\sqrt{\ell_2^2 + r^2}} - \frac{\ell_1}{\sqrt{\ell_1^2 + r^2}}\right]\hat{\phi}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi r}\left[\sin\theta_2 - \sin\theta_1\right]\hat{\phi}$$



2. For the second version of the equation, the one with the sin's, show what happens to the equation for an infinitely long wire. What values will you use for the θs?

3. Find the magnetic field at the center of a square loop, which carries a steady current I . Let R be the distance from center to side (Fig. 5.22 from Griffith's text).

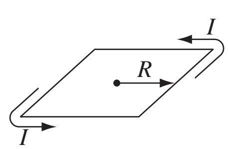
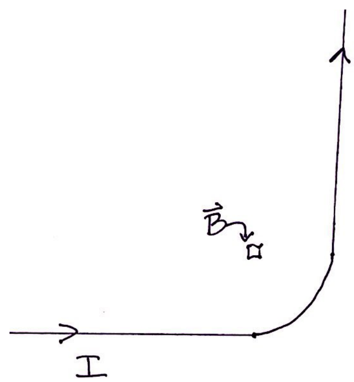


FIGURE 5.22

4. An infinitely long wire has been bent into a right angle turn, as shown in the figure. The "curvey part" where it bends is a perfect quarter circle, radius R. A steady current I flows through this wire. Find the magnetic field at the center of that quarter circle (marked by my usual empty box).



Ampere's Law Practice

Ampere's Law is:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{thru}$$

Learning how to choose an Amperian Loop is a skill we should practice.

5. Imagine there is a constant magnetic field $\vec{B} = 2T\hat{y}$ as shown by the (solid) field lines below. An Amperian loop is also shown below (dashed lines). Let's say the rectangular loop has dimensions $6cm \times 3cm$.

   (a) What is $\int \vec{B} \cdot d\vec{\ell}$ for each side of the loop?

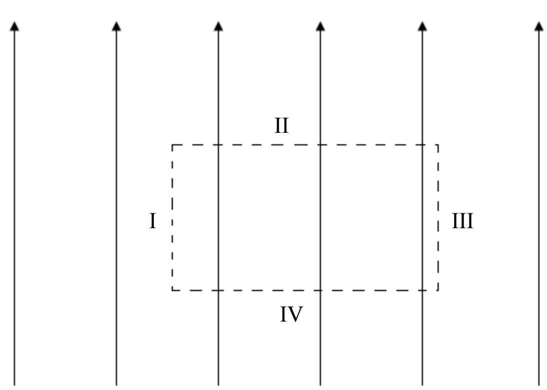
      i. Side I:

      ii. Side II:

      iii. Side III:

      iv. Side IV:

   (b) What is the total, $\oint \vec{B} \cdot d\vec{\ell}$?



6. Now imagine rotating the Amperian loop such that it makes an angle $\theta = 30°$ with respect to the magnetic field (shown below).

   (a) What is $\int \vec{B} \cdot d\vec{\ell}$ for each side of the loop?

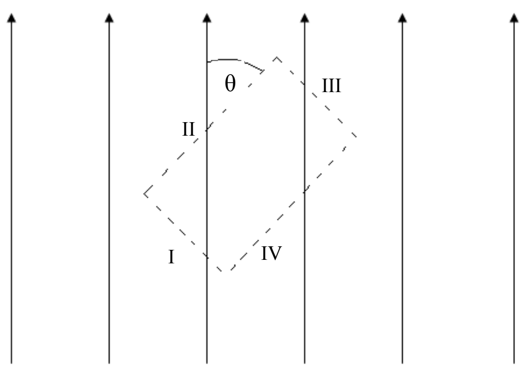
      i. Side I:

      ii. Side II:

      iii. Side III:

      iv. Side IV:

   (b) What is $\oint \vec{B} \cdot d\vec{\ell}$?



7. Compare $\oint \vec{B} \cdot d\vec{\ell}$ for the two previous cases. Do they make sense? Explain.

8. Thinking about Problems 5 and 6:

   (a) Qualitatively explain how your results for questions 5 and 6 would change if your Amperian loop was a circle instead of a rectangle.

   (b) Why is a rectangular Amperian loop better for this problem than a circular Amperian loop? Explain.

   (c) What sort of situation might you want a circular Amperian loop for and why? Be explicit.

9. If for an Amperian loop, $\oint \vec{B} \cdot d\vec{\ell} = 0$ (not necessarily the one in questions 5 and 6), can you conclude anything about the magnetic field B? Explain.

10. What does it mean if $\oint \vec{B} \cdot d\vec{\ell} \neq 0$? Explain.