FINAL EXAM REVIEW

kinematics

position, $\vec{r} = \begin{bmatrix} x \\ y \end{bmatrix}$ - in m
- vector

velocity, $\vec{v} \equiv \frac{d\vec{r}}{dt} = \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \end{bmatrix}$
- m/s
- vector

acceleration $\vec{a} \equiv \frac{d\vec{v}}{dt} = \begin{bmatrix} a_x \\ a_y \end{bmatrix} = \begin{bmatrix} \frac{dv_x}{dt} \\ \frac{dv_y}{dt} \end{bmatrix}$
- m/s^2
- vector

special case $\vec{a} = \text{const} = \vec{a}_0$

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$v^2 = v_0^2 + 2\vec{a}\cdot\Delta\vec{r}$$

Recall you can write these as col. vectors

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} v_{0x} \\ v_{0y} \end{bmatrix} + \begin{bmatrix} a_x t \\ a_y t \end{bmatrix}$$ ← Each row is a 1D kinematic eq.

Displacement, $\Delta\vec{r} \equiv \vec{r}_f - \vec{r}_i$

- in m
- vector

$\Delta \equiv$ change in (usually final - initial)

All the <u>averages</u> get def'd this way

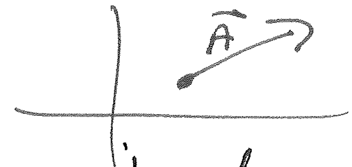
$$\vec{v}_{ave} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{a}_{ave} = \frac{\Delta\vec{v}}{\Delta t}$$

$$P_{ave} = \frac{\Delta W}{\Delta t} = \frac{\Delta KE}{\Delta t} = \frac{\Delta PE}{\Delta t} \quad \ldots$$

units Watt $= \frac{J}{s}$

Vectors



$\equiv$ quantity with magnitude & direction

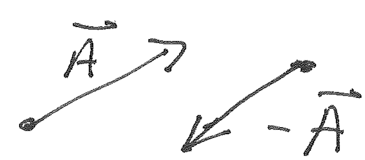
ex: position, displacement, velocity, acc force, momentum, ang. mom., torque, ...

<u>scalar</u> $\equiv$ quantity w/ magnitude only

ex: distance, speed, mass, temp,

Vector rules

1. Negation
   - flip the vector by 180°



   - negate all comp.s

$$\vec{A} = \begin{bmatrix} A_x \\ A_y \end{bmatrix} \qquad -\vec{A} = \begin{bmatrix} -A_x \\ -A_y \end{bmatrix}$$

2. Addition

- put them tail-to-tip, sum goes from tail of first vector to tip of final vector in sum.

- Add like componets.

$$\vec{C} = \vec{A} + \vec{B}$$

$$\begin{bmatrix} C_x \\ C_y \end{bmatrix} = \begin{bmatrix} A_x \\ A_y \end{bmatrix} + \begin{bmatrix} B_x \\ B_y \end{bmatrix}$$

3. Subtraction

= Negate vector(s) to be subtracted then add.

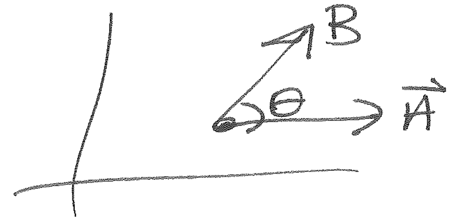
4. Mult by a scalar

= scale length of vector by scalar

= mult each comp by scalar

ex $2\vec{A} = \begin{bmatrix} 2A_x \\ 2A_y \end{bmatrix}$

5. Dot prod / Scalar product



$$C = \vec{A} \cdot \vec{B}$$
$$= AB\cos\theta$$
$$= A_x B_x + A_y B_y$$

ex: Work $W = \vec{F} \cdot \vec{d}$

6. Vector Product / Cross Product

$$\vec{C} = \vec{A} \times \vec{B}$$
$$|\vec{C}| = C = AB\sin\theta$$

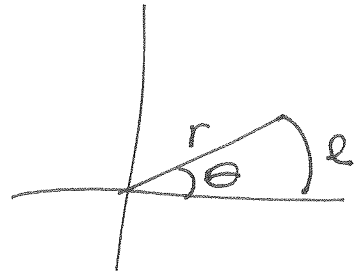
dir of $\vec{C}$ from Right Hand Rule

ex: Torque, $\vec{\tau} = \vec{r} \times \vec{F}$

ang mom, $\vec{L} = \vec{r} \times \vec{p}$

Rotational kinematics

ang position, θ



arc length $\ell = r\theta$

- units radians
- is a vector (we use cw or ccw)

ang velocity, ω

$$\omega \equiv \frac{d\theta}{dt}$$

- vector
- $\text{rad}/s = 1/s$

ang acceleration, α

$$\alpha \equiv \frac{d\omega}{dt}$$

- vector
- $\text{rad}/s^2 = \frac{1}{s^2}$

for const r (circ motion rot motion)

$$\ell = r\theta$$

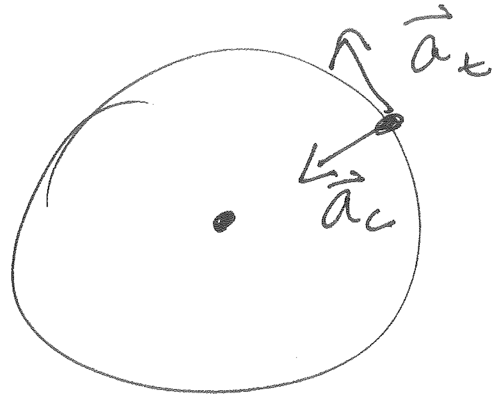
take deriv

$$v_\ell = r\omega$$

take deriv

$$a_\ell = r\alpha$$

$\vec{a}$ in tangential direction. (along arc length)



$\vec{a}$ can have a comp along motion <u>and</u> toward center.

$$a_t = r\alpha \qquad a_c = \frac{v^2}{r}$$

Rotational kinematic eq's (const $\vec{\alpha}$)

$$\vec{\theta} = \vec{\theta}_0 + \vec{\omega}_0 t + \frac{1}{2}\vec{\alpha} t^2$$

$$\vec{\omega} = \vec{\omega}_0 + \vec{\alpha} t$$

$$\omega^2 = \omega_0^2 + 2\vec{\alpha} \cdot \Delta\vec{\theta}$$

<u>Force</u>, $\vec{F}$ ≡ push or pull

- vector

- units $N = kg \frac{m}{s^2}$

Newton's Three Laws

1. An object in <u>uniform motion</u> remains in uniform motion unless acted on by a net, external force.

2. $\Sigma \vec{F} = m\vec{a}$

3. $\vec{F}_{12} = -\vec{F}_{21}$

Forces we learned

<u>Gravity</u>, $\vec{F}_g = m\vec{g}$ (acts at c-m)

<u>Normal force</u>, $\vec{F}_N$ always $\perp$ to surface

must use 2nd Law to find it.

<u>Tension</u>, $\vec{F}_t$ dir = along "string" away from object

use 2nd Law to find.

Forces (cont.)

friction, $\vec{F}_{fr}$

dir:

mag: $F_{fr} = \mu F_N$

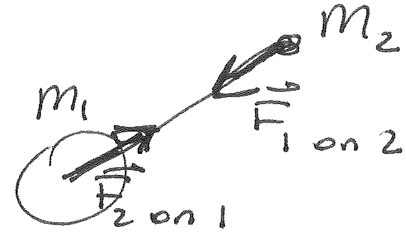
μ = const dep on materials

Spring Force (Hook's Law), $\vec{F}_s$

$F_x = -kx$ $x \equiv$ dist from equil

Universal Grav

$$F_G = G\frac{m_1 m_2}{r^2}$$



dir: along line between them, toward other

Buoyant Force, $\vec{F}_B$

$F_B = m_f g$ m_f = (mass of fluid displaced)

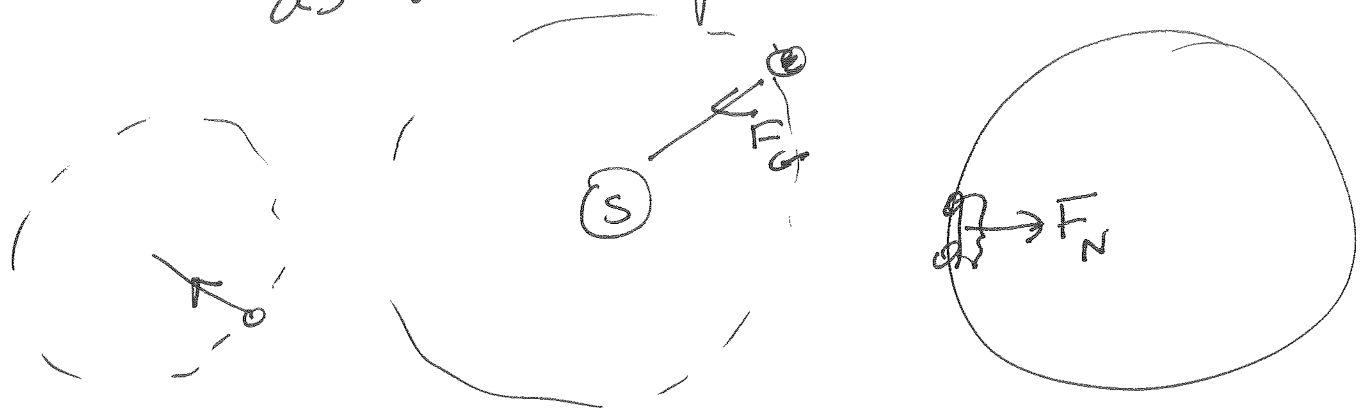
Free body diagrams!

Circular motion

$$a_c = \frac{v^2}{r} \Leftarrow \text{toward the center}$$

centripetal is a <u>direction</u>

we have seen many forces act as a <u>centripetal</u> force



F_T Tension

F_G grav.

Normal

Work, W

- scalar
- units Joules, $J = Nm$

Best def: $W \equiv \int \vec{F} \cdot d\vec{r}$

if $\vec{F}$ is const ← (not springs, not univ. grav)

$W = \vec{F} \cdot \Delta\vec{r}$

↑ book uses $\vec{d}$

$W = F d \cos\theta = F_{\parallel} d$

Conservative forces ≡ those for which work done by the force is path ind.

ex: gravity (both forms)
spring force

Work - Kinetic Energy Thm

$$W_{net} = \Delta KE$$

$$KE \equiv \frac{1}{2} m v^2$$

- scalar
- units, J

[later also $KE_{rot} = \frac{1}{2} I \omega^2$]

Potential Energy, PE

$$\Delta PE_c \equiv -W_c$$

$$PE_g = mgh$$

$$PE_s = \frac{1}{2} k x^2$$

Cons of Energy

$$E \equiv KE + PE$$

← Total mech. energy, E

Cons of E

$$\boxed{E_i + W_{nc} = E_f}$$

if $W_{nc} = 0$

$$\boxed{E_i = E_f}$$

Momentum, $\vec{p}$

$\vec{p} \equiv m\vec{v}$

- vector
- units $kg \frac{m}{s}$

Rewrite 2nd Law: $\Sigma \vec{F} = \frac{d\vec{p}}{dt}$

$\vec{p}$ is cons if no net, ext force

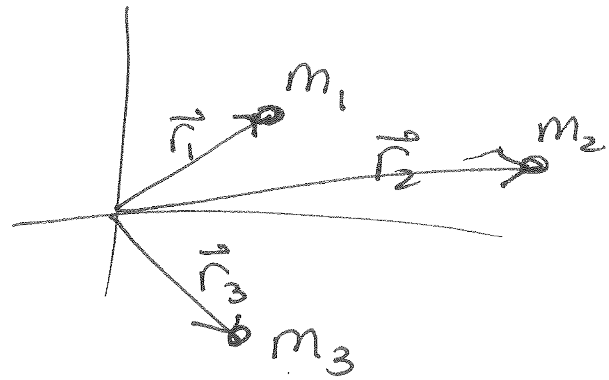
Collisions

$\vec{p}_{tot}$ is conserved (include all objects that "collide")

KE may or may not be cons.

Center of mass

$$\vec{R}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

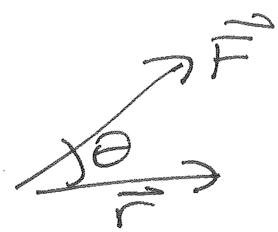


Rotational Motion

kinematics

dynamics

Torque, $\vec{\tau} \equiv \vec{r} \times \vec{F}$



- vecter
units m·N

$$\boxed{\tau = rF\sin\theta}$$

$$\tau = r_\perp F$$

dir: CW or CCW is fine

(also Right Hand Rule)

Moment of Inertia & 2nd Law for Rotations

$$\boxed{\sum \vec{\tau} = I\vec{\alpha}}$$

where $\boxed{I = \sum_i m_i r_i^2}$

for point pts

I = look up on table for solid objects (given on exams)

Angular momentum, $\vec{L}$

$$\vec{L} \equiv \vec{r} \times \vec{p}$$

for circ motion, $L = rmv$

$$L = rp\sin\theta$$

$$\boxed{\vec{L} = I\vec{\omega}}$$

Rotational 2nd Law and Cons of $\vec{L}$

$$\Sigma \vec{\tau} = \frac{d\vec{L}}{dt}$$

if $\Sigma \vec{\tau} = 0$

$\vec{L} = \text{const}$ (= conserved)

$$\boxed{\vec{L}_i = \vec{L}_f}$$

Fluids

density, ρ $\qquad \rho \equiv \frac{M}{V} = \frac{\text{mass}}{\text{volume}}$

pressure, $P \equiv \frac{F}{A} = \frac{\text{force}}{\text{area}}$

units $= \frac{N}{m^2}$

gauge pressure in fluid

$P = \rho g h$

absolute Pressure

$$P = P_{atm} + P_{gauge}$$

Buoyant force, $F_B = m_f g$

m_f = mass of fluid displaced

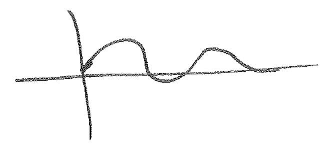
Simple Harmonic Motion

Def: $F \propto -x$

$$F = -m\omega^2 x$$

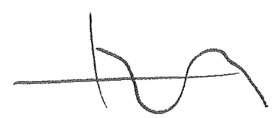
Has solutions:

$$x = A\sin(\omega t)$$



or

$$x = A\cos(\omega t)$$



Cases:

Spring: $F = -kx$

$$\omega^2 = \frac{k}{m}$$

Simple Pendulum: (for small θ)

$$\omega^2 = \frac{g}{l}$$

$$v_{max} = \omega A$$

$$E = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2 = \frac{1}{2}m\omega^2A^2$$

Waves

$\equiv$ transport energy & momentum w/o transferring <u>matter</u>

each "piece" of medium oscillates around an equilibrium position (could be SHM)

$$y = A\sin(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

$\lambda \equiv$ wavelength

= distance between repeating patterns

speed of any wave

$$v = \lambda f = \frac{\lambda}{T}$$

speed of wave on string

$$v = \sqrt{\frac{F_T}{\mu}}$$

where $\mu = \frac{\text{mass}}{\text{length}}$

Standing waves

Rope fixed at both ends

$L = \lambda/2$

$L = \lambda$

$L = \frac{3\lambda}{2}$

⋮

$L = \frac{n\lambda}{2}$

$$v = \lambda f$$

$$f = \frac{v}{\lambda} = \frac{v}{\frac{2L}{n}}$$

$$\boxed{f_n = n \frac{v^*}{2L}}$$

n = # of antinodes
= # of harmonic

nodes ≡ points remain still

antinodes ≡ constructive interference

Superposition of waves
(They add alegebraically)