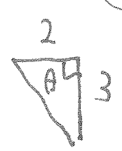
1. (8 points) Physicists try to find mathematical models for predicting the behavior of natural phenomena. In doing so, we define physical concepts in terms of mathematical quantities such as time, distance, displacement, velocity, speed, acceleration, mass, weight and force.

   List 4 physical quantities which are vectors.

   Displacement Acceleration Weight
   Velocity Force

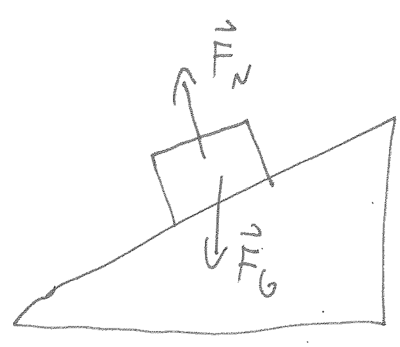
2. (10 points) A vector is given in column notation as $\vec{A} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ Write the vector as magnitude and direction. (Don't forget to show your work.)

   $|\vec{A}| = \sqrt{4+9} = 3.61$



   $\theta = \tan^{-1}\left(\frac{3}{2}\right) = 56.3°$ below +x axis

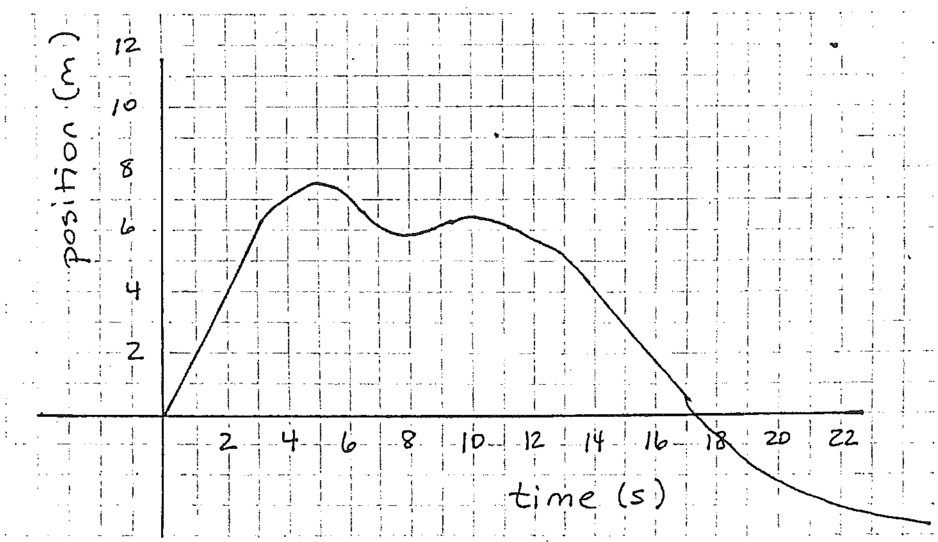
3. (6 points) A book slides down a smooth incline. (Friction is negligible.) The incline makes an angle of 25° relative to the horizontal. Draw a free body diagram for the book.



4. (6 points) Is it possible for an object to have velocity and acceleration in opposite directions? If yes, give a specific example. If no, explain why not.

   yes, throwing a ball upward after release
   (upward $\vec{v}$, downward $\vec{a}$)

5. (15 points) Bee gone prospecting. The graph below represents the position of a honey bee as it travels back and forth on a straight line. As you read values from the graph to answer the questions, there will some uncertainty in the tenths digits. I recognize this and will mark accordingly.



   (a) Identify two times at which the bee's velocity is zero. 5 s, 8 s, 10 s

   (b) What is the bee's velocity at $t = 2$ s?

   $\vec{v}(2 \text{ seconds}) \approx \frac{\Delta \vec{x}}{\Delta t} = \frac{6\text{ m} - 2\text{ m}}{3\text{ s} - 1\text{ s}} \approx 2\text{ m/s}$

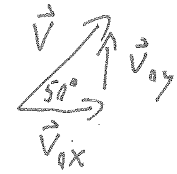
6. (15 points) A physics professor throws a pen into the air with an initial velocity of 4m/s at an angle of 50° relative to the horizontal. (Imagine the prof was crouched down and threw it from ground level.)

   (a) How long does it take for the pen to get to the highest point?

   (b) What is the pen's velocity right before it hits the ground?

a)

$\vec{V}_{0y} = (4\,m/s)\sin 50° = 3.064\,m/s$



$\vec{V}_y = \vec{V}_{0y} + \vec{a}_y t$

$0 = 3.064\,m/s + (-9.8\,m/s^2)\,t$

$t = 0.313\,s$ (0.306 if used $\vec{g} = -10\,m/s^2$)

b) Since ground is level: $\vec{V}_y = -\vec{V}_{0y} = -3.064\,m/s$

Also $\vec{V}_x = \vec{V}_{0x} = (4\,m/s)\cos\theta = 2.571$

$\vec{V} = \begin{pmatrix} 2.571 \\ -3.064 \end{pmatrix}$ m/s $\quad |\vec{V}| = \sqrt{V_x^2 + V_y^2} = 4\,m/s \quad \theta = \tan^{-1}\left(\frac{-3.064}{2.571}\right) = -50°$

or

$\vec{V} = 4\,m/s$, 50° below +x axis

(Can also calculate this:

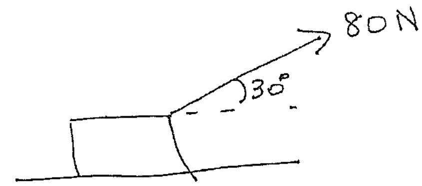
$V_y^2 = V_{0y}^2 + 2a_y(y - y_0)$ ← $0 = y - y_0$

$= V_{0y}^2$

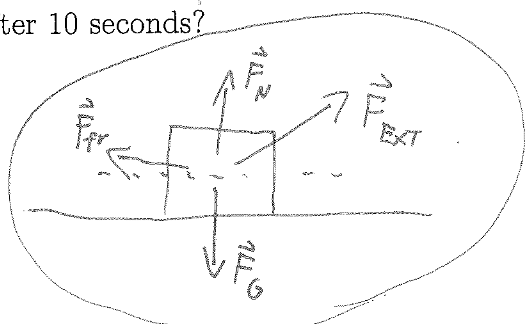
$V_y = -V_{0y}$

7. (25 pts) A crate is pulled across the *rough* floor of a warehouse with a force of 80 N at an angle of 30° above the horizontal. The box has a mass of 15 kg and accelerates at a rate of 2 m/s^2.

   (a) Draw a free-body diagram for the crate.
   (b) What is the magnitude of the friction force acting on the crate?
   (c) What is the coefficient of kinetic friction between the crate and the floor?
   (d) If the crate starts from rest, how far does it travel after 10 seconds?



a)



$$\vec{F}_{EXT} = \begin{pmatrix} (80N)\cos 30° \\ (80N)\sin 30° \end{pmatrix} = \begin{pmatrix} 69.3\ N \\ 40\ N \end{pmatrix}$$

b) $\sum \vec{F} = m\vec{a}$ $\qquad \vec{F}_{EXT} + \vec{F}_N + \vec{F}_G + \vec{F}_{fr} = m\begin{pmatrix} 2\ m/s^2 \\ 0 \end{pmatrix}$ $\qquad \vec{a}$ is in x-direction only

$$\begin{pmatrix} 69.3\ N \\ 40.0\ N \end{pmatrix} + \begin{pmatrix} 0 \\ F_N \end{pmatrix} + \begin{pmatrix} 0 \\ m\vec{g} \end{pmatrix} + \begin{pmatrix} F_{fr} \\ 0 \end{pmatrix} = \begin{pmatrix} 30\ N \\ 0 \end{pmatrix}$$

$$F_{fr} = 30\ N - 69.3\ N = -39.3\ N \qquad |\vec{F}_{fr}| = 39.3\ N$$

c) $$\mu_k = \frac{|\vec{F}_{fr}|}{|\vec{F}_N|}$$

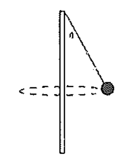
$$F_N = 0 - 40\ N - 15kg(-9.8\ m/s^2) = 107\ N$$

$$\mu_k = \frac{39.3\ N}{107 N} = 0.367 \qquad (0.357 \text{ if used } \vec{g} = -10\ m/s^2)$$

d) $$\Delta\vec{x} = v_0 t + \frac{1}{2}\vec{a}_x t^2$$

$$= 0 + \frac{1}{2}(2\ m/s^2)(10 s)^2 = 100\ m$$

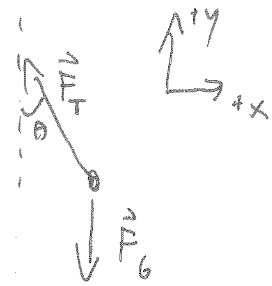
8. (15 pts) A child on a playground hits a tether ball of mass 0.5kg so that it moves in a circle around the pole with a constant speed of 2m/s. If the tether (rope attached to the ball) makes an angle of 30° with the pole:



(a) What is the tension in the rope holding the ball?

(b) What is the radius of the circular path of the ball (as it moves around the pole)?

a)



$$\Sigma \vec{F} = \vec{F}_T + \vec{F}_G = m\vec{a}$$

$$= \begin{pmatrix} -|\vec{F}_T| \sin\theta \\ |\vec{F}_T| \cos\theta \end{pmatrix} + \begin{pmatrix} 0 \\ m\vec{g} \end{pmatrix} = m \begin{pmatrix} \vec{a}_x \\ \vec{a}_y \end{pmatrix} \qquad \vec{a}_y = 0$$

$$|\vec{F}_T| \cos\theta = -m\vec{g} = 4.9\,N$$

$$|\vec{F}_T| = 5.66\,N \qquad (5.77\,N \text{ if used } \vec{g} = -10\,m/s^2)$$

$$\vec{F}_T = \begin{pmatrix} -2.83\,N \\ 4.9\,N \end{pmatrix} \quad \text{or} \quad |\vec{F}_T| = 5.66\,N \text{ at } 30° \text{ from vertical}$$

b. $|\Sigma \vec{F}_R| = m\frac{v^2}{r}$ $\qquad |\Sigma \vec{F}_R| = |-|\vec{F}_T| \sin\theta| = 2.83\,N$

$$r = \frac{mv^2}{|\Sigma \vec{F}_R|} = 0.707\,m$$