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Physics 10

Fall 2017

Exam 2

The Ground Rules: Show your work! You must give sufficient justification for your answer–no credit will be given for answers that are unaccompanied by an explanation and/or clearly written calculations. Where required, answers must include the correct units and direction. Please keep 3 sig figs in your answer for numerical problems.

Point values are shown next to the problem; there are a total of 100 points on the exam.

Some constants:
$g = 9.8$ m/s^2 and $G = 6.67 \times 10^{-11} Nm^2/kg^2$.

Some useful formulas

From Exam 1:

$$\Delta \vec{r} \equiv \vec{r}_f - \vec{r}_i$$

$$\vec{v}_{AVE} \equiv \frac{\Delta \vec{r}}{\Delta t}$$

$$\vec{v} \equiv \frac{d\vec{r}}{dt}$$

$$\vec{a}_{AVE} \equiv \frac{\Delta \vec{v}}{\Delta t}$$

$$\vec{a} \equiv \frac{d\vec{v}}{dt}$$

$$\vec{v} = \vec{v_0} + \vec{a}t$$

$$\vec{r} = \vec{r_0} + \vec{v_0}t + \frac{1}{2}\vec{a}t^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\Sigma \vec{F} = m\vec{a}$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

$$\vec{F}_G = m\vec{g}$$

$$F_{fr} = \mu F_N$$

$$a_c = \frac{v^2}{r}$$

New on Exam 2

$$F = G\frac{Mm}{r^2}$$

$$W \equiv \vec{F} \cdot \vec{d} = Fd_{//} = Fd\cos\theta$$

$$W_{net} = \Delta KE$$

$$KE \equiv \frac{1}{2}mv^2$$

$$\Delta PE \equiv -W_c$$

$$PE_g = mgh \qquad PE_s = \frac{1}{2}kx^2$$

$$F_s = -kx$$

$$E \equiv KE + PE$$

$$E_i + W_{nc} = E_f$$

$$P \equiv \frac{dW}{dt} \qquad P_{ave} \equiv \frac{\Delta W}{\Delta t}$$

$$\vec{p} \equiv m\vec{v}$$

$$\Sigma \vec{F} = \frac{d\vec{p}}{dt} \qquad \Sigma \vec{F}_{ave} = \frac{\Delta \vec{p}}{\Delta t}$$

$$X_{cm} = \frac{\Sigma x_i m_i}{M}$$

$$a_t = r\alpha \qquad v = r\omega$$

$$\vec{\omega} = \vec{\omega}_0 + \vec{\alpha}t$$

$$\vec{\theta} = \vec{\theta}_0 + \vec{\omega}_0 t + \frac{1}{2}\vec{\alpha}t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$I \equiv \Sigma m_i r_i^2$$

$$KE = \frac{1}{2}I\omega^2$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = rF\sin\theta = r_\perp F$$

$$\Sigma \vec{\tau} = I\vec{\alpha}$$

$$L = r_\perp p = rp\sin\theta$$

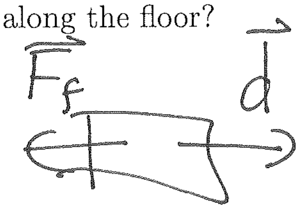
$$L = I\omega$$

1. (7 points) What would happen to the gravitational force between two objects if the distance between them were doubled?

$$F_0 = G\frac{m_1 m_2}{r_0^2} \Longrightarrow G\frac{m_1 m_2}{(2r_0)^2}$$

$$\boxed{\tfrac{1}{4} \text{ as much}}$$

2. (8 points) A crate sliding across a rough, horizontal floor has an initial energy of 20J. As the crate slides, it experiences a friction force of magnitude 4N. What is the object's energy after it slides 3m along the floor?

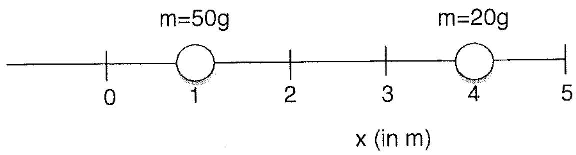


$$E_f = E_i + W_{nc}$$

$$= 20\,J - 4\,N\,(3m)$$

$$= \boxed{8\,J}$$

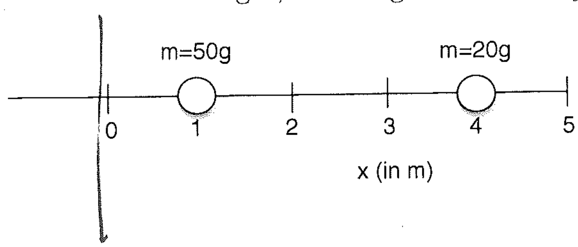
3. (7 points) Where is the center of mass of the configuration shown here?



$$X_{cm} = \frac{50g(1m) + 20g(4m)}{70g} = \frac{5\cancel{0}+8\cancel{0}}{7\cancel{0}} = \frac{13}{7}$$

$$\boxed{X = 1.86\,m}$$

4. (8 points) What is the moment of inertia of the configuration shown here? (The axis of rotation is the origin, rotating around the y-axis.)



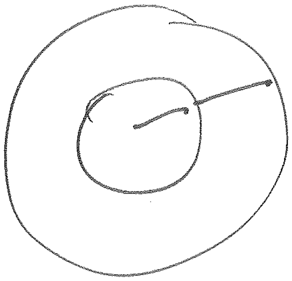
$$I = \sum m_i r_i^2 = (.050g)(1m)^2 + (.02kg)(4m)^2$$

$$= .05 + .32$$

$$= \boxed{.37\ kg\ m^2}$$

5. (10 pts) A satellite orbits the earth at a height of one earth radius above the surface of the earth. (The radius of the earth is $R_E = 6 \times 10^6$m, and the mass of the earth is $M_E = 6 \times 10^{24}$kg.) If the orbit is a uniform circle, what is the magnitude and direction of the satellite's acceleration?

$$F_s = G\frac{m_e \cancel{m_s}}{(2R_e)^2} = \cancel{m_s} a_s$$



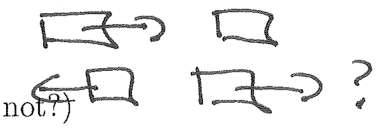
$$a = \left(6.67\times10^{-11}\,\frac{Nm^2}{kg^2}\right)\frac{(6\times10^{24}\,kg)}{(2\cdot6\times10^{6}\,m)^2}$$

$$\boxed{a = 2.78\,m/s^2 \text{ toward the center of earth}}$$

6. (15 pts) A 500g cart moving to the right with speed 8m/s hits a second cart of mass 1kg at rest. After the collision, the 500g cart bounces back with a speed of 2 m/s to the left,

(a) What is the velocity of the 1kg cart after the collision?

(b) Is the collision elastic or inelastic? (Does it conserve energy or not?)



a.) $m_1\vec{v}_1 = m_1\vec{v}_1' + m_2 v_2'$

$\cancel{m}(8\frac{m}{s}) = \cancel{m}(-2\frac{m}{s}) + 2\cancel{m}\, v_2'$

-8 if they assume cons of E

$10\frac{m}{s} = 2v_2'$

$\boxed{v_2' = 5\,m/s \text{ right}}$

b.) $\frac{1}{2}m_1 v_1^2 \overset{?}{=} \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2$

$m(8)^2 \overset{?}{=} m(-2)^2 + 2m(5)^2$

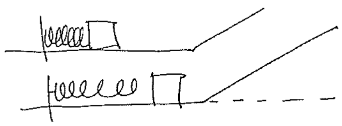
$64 \overset{?}{=} 4 + 2(25)$

$64 \neq 54$

$\boxed{\text{Inelastic}}$

KE not cons!

7. (15 pts) A block of mass 2kg is connected to a spring with spring constant $k = 300N/m$. The spring is compressed horizontally $d = 15$cm and then released.



(a) Find the speed of the block as it passes through equilibrium

(b) If the block leaves the spring at equilibrium and then slides up a frictionless incline, how high will the block go?

a.) $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$v = \sqrt{\frac{k}{m}}\,x = \sqrt{\frac{300\frac{N}{m}}{2kg}}\,(.15m)$

$= \boxed{1.84\ m/s}$

b.) $\frac{1}{2}kx^2 = mgh$

$h = \frac{kx^2}{2mg} = \frac{(300\frac{N}{m})(.15m)^2}{2(2kg)(9.8\frac{m}{s^2})}$

$\boxed{h = 17.2\ cm}$

8. (10 points) A bicyclist traveling at 10 m/s brakes (with no slipping) to avoid a deer crossing the bike trail. The bike decelerates uniformly to rest at 2 m/s^2. The wheels of the bike have a radius of 50 cm. How many revolutions does one of the bicycle wheels make during braking?

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

or

$$v = v_0 + at$$

$$t = 5s$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$x = (10 m/s)(5s) - \frac{1}{2}(2\frac{m}{s^2})(5s)^2 = 25m$$

or

$$v^2 = v_0^2 + 2a(x - x_0)$$

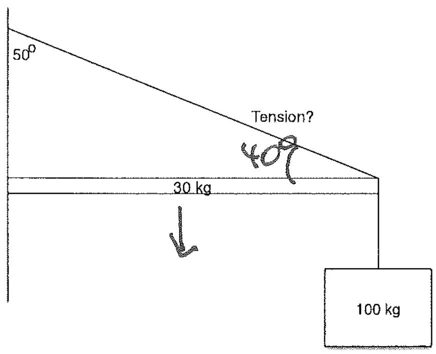
$$0 = (10\frac{m}{s})^2 - 2(2 m/s^2)\ x$$

$$x = \frac{100}{4} = 25m$$

$$x = r\theta \qquad \theta = \frac{x}{r} = \frac{25\cancel{m}}{.5\cancel{m}} = 50 \text{ rad}$$

$$\theta = 50 \cancel{rad} \times \frac{1 \text{ rev}}{2\pi \cancel{rad}} = \boxed{7.96 \text{ rev}}$$

9. (20 pts) A store owner would like to hang a sign from the end of a beam of mass 30kg and length 3m. The beam is held to the wall by a supporting wire which makes a 50° angle with the supporting wall as shown below. The sign has a mass 100kg. (The moment of inertia for a beam around one end is $I = \frac{1}{3}ML^2$.)



(a) What is the tension in the supporting wire? (The wire attached to the wall.)

(b) If the supporting wire suddenly breaks, what is the angular acceleration of the beam (around the end attached to the wall) at that instant when the beam is still horizontal?

a.) $\Sigma \tau = 0$

$$0 = (m_b g)\frac{\cancel{\ell}}{2}\sin 90° + (m_s g)\cancel{\ell}\sin 90° - T\cancel{\ell}\sin 40°$$

$$T = \frac{(\frac{m_b}{2} + m_s)g}{\sin 40°} = \frac{(1\overset{15}{\cancel{30}}\text{ kg})\,9.8\text{ m/s}^2}{\sin 40°}$$

$$\boxed{T = \cancel{1982}\ 1753\text{ N}}$$

b.) $\Sigma \tau = I\alpha$

$$\left(m_b\frac{\cancel{\ell}}{2} + m_s\cancel{\ell}\right)g = \left(\frac{1}{3}m\ell^{\cancel{2}}\right)\alpha$$

$$\alpha = \frac{3g\left(\frac{m_b}{2} + m_s\right)}{m_b \ell} = \frac{3(9.8\frac{m}{s^2})[115\text{ kg}]}{(30\text{ kg})(3\text{ m})} = \boxed{37.6\ \frac{\text{rad}}{s^2}\ \text{CW}}$$

b.) $\Sigma \tau = I\alpha$

$$(m_b g)\frac{\ell}{2} + \overset{T_s}{\cancel{m_s g}}\,\ell = \left(\frac{1}{3}ML^2\right)\alpha$$

$$\Sigma F_s = m a_s$$

$$T - m_s g = -m_s a$$

$$T = m_s (g - a)$$

$$\quad = m_s g - m_s \ell \alpha$$

$$m_b g \frac{\ell}{2} + \left[m_s g \ell - m_s \ell^2 \alpha\right] = \frac{1}{3} M_b \ell^2 \alpha$$

$$m_b g \frac{\cancel{\ell}}{2} + m_s g \cancel{\ell} = \left(\frac{m_b}{3} + m_s\right)\ell^{\cancel{2}} \alpha$$

$$\alpha = \frac{\left(\frac{m_b}{2} + m_s\right) g}{\left(\frac{m_b}{3} + m_s\right)\ell} = \frac{(15 + 100)}{(10 + 100)} \frac{9.8 \frac{m}{s^2}}{3m} = 3.42 \frac{rad}{s^2}$$