9/8 In Class–Some Calculus and 1D Kinematics

Review of differentiation with position, velocity, and acceleration as the examples

Taking derivatives of polynomials and trig functions is about the extent of what you'll need to do in terms of derivatives. Let's see what you remember!

For each of the following, x represents the position of a particle as a function of time. Make a quick sketch of x vs t, then take the derivative and sketch v vs t. Note: Writing units in *symbolic* equations can get very confusing, especially in handwriting. For example, is that m a "meter" or does it mean the mass? Whenever it doesn't state otherwise, the default is that constants have whatever units they need for v, x, and t to work out to m/s, m, and s respectively.

(I really do mean quick sketch. I'm interested in the shape of the curve more than numbers.)

Keep track of your $v(t)$'s, you will need them again.

1. $x = 3$
2. $x = 3t$
3. $x = 3t + 5t^2$
4. $x = \sin 2\pi t$
5. $x = \cos 2\pi t$

acceleration

acceleration, a $\equiv$ time rate of change of *velocity*, in other words, the first derivative of velocity wrt time.

$$\vec{a} \equiv \frac{d\vec{v}}{dt}$$

6. For all of the velocities you found in 1-5, find the accelerations of the particle. Sketch a vs t for 3-5.

Review integration with acceleration, velocity and position as the examples

Can you go backwards? Recall that the integral is the inverse operation of the derivative. Thus,

$$\vec{v} = \int \vec{a}dt \qquad \vec{x} = \int \vec{v}dt$$

For each of the following, given the acceleration, find the velocity and then the position. For all of these, you may assume the particle starts from rest from the origin.

7. $a = 0$
8. $a = 5$
9. $a = 4t$

10. $a = \cos \pi t$

Special case: constant acceleration,
or, Derive two kinematic equations

Some notation: in physics, we often use a subscript of zero to denote the quantity at time $t = 0$. For example, x_0 is the position at time $t = 0$. (Both x and $x(t)$ mean the position at any time t.) Similarly, v_0 is the velocity at time $t = 0$ and a_0 is the acceleration at time $t = 0$.

11. If the acceleration of a particle is constant, we often call it $a_0 = a$. (The acceleration at any time (a) is the same as the initial acceleration (a_0).) Find an expression for the velocity as a function of time *if the particle starts from rest at the origin.* (Hint: This is just like problem 8 above, but with a letter rather than a number.)

12. If the acceleration of a particle is constant, find an expression for the velocity as a function of time if the particle starts with some initial velocity v_0 and some initial position x_0. Write the constant of integration in terms of v_0. (Hint: plug in $t = 0$ to your expression for v.)

13. Use your answer to the last problem, integrate one more time to get position as a function of time. Again, write your constant of integration in terms of x_0.

    You just derived two kinematic equations. (Some books derive a third and fourth, but they are pure algebra, solving one for t and plugging into the other.)

    It turns out that **the acceleration of every object in free fall (near the surface of the earth) is 9.8 m/s^2 toward the center of the earth (down).** It turns out, this is the gravitational field near the surface of the earth. Thus, we use the symbol, $\vec{g}$ for what I will call "the acceleration due to gravity near the surface of the earth." Many of you will want to say "gravity" for short. But that's incorrect–gravity is a force, and this is a field which turns out (in this ONE case) to be the same as the acceleration. You can say it, we're all lazy, but remember that it is not a force!

    Given $g = 9.8 m/s^2$ (down),

14. An object is dropped (dropped means no initial velocity) from a height of 10m above the ground.

    (a) Using the position equation you derived in the last problem, can you solve for the time to impact?

    (b) Now plug that time into the velocity equation and solve for the speed of the object just before it hits the ground.