10/11 In Class–Work, Energy, and Conservation of Energy

Summary of Work and Energy so far....

$$W = \vec{F} \cdot \vec{d} = F_{//}d = Fd\cos\theta = F_x d_x + F_y d_y$$

Energy $\equiv$ ability to do work. Both work and energy are measured in Joules. Both are scalars, meaning they have no direction. They can be positive or negative, and that matters.

Kinetic Energy is the ability to do work because of motion.

$$KE = \frac{1}{2}mv^2$$

Recall that it makes sense to ask about the work done by any particular force, and something interesting is true for the total work:

$$Wnet = \Delta KE$$

Potential Energy is the ability to do work because of position.

$$\Delta PE_f \equiv -W_f$$

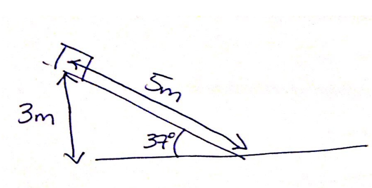
$$PE_g = mgh$$

1. A block of mass 4kg slides down a smooth (frictionless) incline. The incline makes an angle of 37° wrt the horizontal. The block slides a distance of 5m down the incline, over a vertical height of 3m. (Notice that $\sin 37° = 3/5 = \cos 53°$.)

   (a) How much work does gravity do on the block as it slides?

   (b) How much work does the normal force do on the block? (For a frictionless surface, those are the only two forces acting.)

   (c) Use the Work-KE Theorem (3rd equation above) to find the speed just before it hits the ground.



New! : Conservation of Energy

Starting from the Work-KE Theorem:

$$W_{net} = \Delta KE$$

The total work can be split into the work done by conservative forces W_c and the work done by non-conservative forces, W_{nc}. A force is conservative if the work done by that force is path independent. Recall our example of gravity and how the work done by gravity was mgh for both the case where the mass

m fell straight down from height h, and the case where it slid down an incline of height, h. That is what we mean by the work being path independent.

Conservative forces are also the kind for which there is a potential energy defined. Then we have:

$$W_c + W_{nc} = \Delta KE$$

If there are no non-conservative forces, $W_{nc} = 0$, then we have

$$-\Delta PE = \Delta KE$$

$$-(PE_f - PE_i) = KE_f - KE_i$$

where the subscripts i and f mean initial and final.

$$KE_i + PE_i = KE_f + PE_f$$

$$E_i = E_f$$

where E is the total mechanical energy.

$$E \equiv KE + PE$$

$E_i = E_f$ is the statement of Conservation of Energy. Whenever there are no non-conservative forces, the total mechanical energy stays the same. You can use any labels for the points, it does not have to be the "beginning" and the "end." $E_i = E_f = E_1 = E_2 = E_a = E_b$.

2. An book of mass 1.5 kg is dropped from a height of 1.2m above the ground. (If you use g=10m/s^2 you get nice round numbers!)

   (a) What is the book's potential energy at the start?

   (b) What is its kinetic energy at the start?

   (c) What is its total (mechanical) energy at the start?

   (d) If we ignore air resistance, what is the total energy of the book just before it hits the ground?

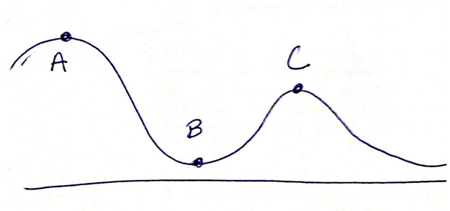
   (e) What is its potential energy just before it hits the ground?

   (f) What is its kinetic energy just before it hits the ground?

   (g) What is its speed just before it hits the ground?

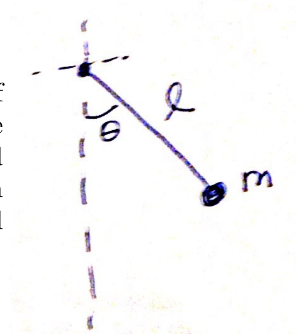
I find it convenient to label the two points, i and f, or 1 and 2, or whatever you want to call them. Then use those points in $E_i = E_f$.

3. **Roller Coaster:** A famous conservation of energy problem is the roller coaster. Only the height matters for the potential energy, so you can find the speed at any point if you know the speed at any one height. For the picture shown here, A is at a height of 20m, B is at 3m, and C is at 10m. If the roller coaster has a speed of 2m/s at A, what is its speed at B and C?



4. **Simple pendulum:** A simple pendulum consists of a mass on the end of a string.

A mass of 1.5 kg swings on the end of a simple pendulum of length 40cm The pendulum bob is released from rest when the string makes an angle of 30° wrt the vertical. Find the speed of the bob as it passes through the center point at the bottom of the arc. You will need a reference point for the potential energy. For now, choose the bottom-most point of the arc.



5. Repeat the last problem using the top of the string, where it connects to the bar, as the reference point for potential energy. What changes? What remains the same?

Springs! We just had time to mention that the spring force is

$$F_s = -kx$$

where x is the displacement from equilibrium.

6. A spring with constant $k = 10N/m$ has an unstretched length (equilibrium length) of 14cm. When you hang a 500g mass from the spring vertically, how much does it stretch when it comes to rest again?

7. Starting from the definition of potential energy,

$$\Delta PE = -W$$

and using the best definition of work,

$$W = \int \vec{F} \cdot d\vec{r}$$

show that for a spring,

$$PE_s = \frac{1}{2}kx^2$$

8. A 2kg mass is connected to a horizontal spring with spring constant of 15N/m. The mass is pulled 25cm from equilibrium and released. How fast it is going when it passed through the equilibrium position?

9. A 2kg mass traveling at a speed of 5m/s across a horizontal, frictionless surface, hits a spring with constant $k = 150N/m$. How much will the spring be compressed?

10. A spring of constant 25N/m is compressed horizontally a distance of 15cm from equilibrium. A mass of 200g is placed at the end of the spring and the spring is released.

    (a) What is the potential energy of the spring when it is compressed 15cm?

    (b) If the surface between the table and the mass is frictionless, how fast will the block be traveling when it leaves the spring? How far will it go?

    (c) If the coefficient of kinetic friction between the block and the table is 0.3, how fast will the mass be traveling when it leaves the spring?

    (d) In the case with friction, how far will the mass travel before coming to rest?