10/13 In Class–Conservation of Energy and when there are Non-conservative forces

This is really a continuation of the last worksheet. I will start from the material you need to start with what is problem 6 on the last worksheet–and I left it problem 6 on this one.

From last time : Conservation of Energy

Starting from the Work-KE Theorem:

$$W_{net} = \Delta KE$$

The total work can be split into the work done by conservative forces W_c and the work done by non-conservative forces, W_{nc}. A force is conservative if the work done by that force is path independent. Recall our example of gravity and how the work done by gravity was mgh for both the case where the mass m fell straight down from height h, and the case where it slid down an incline of height, h. That is what we mean by the work being path independent.

Conservative forces are also the kind for which there is a potential energy defined. Then we have:

$$W_c + W_{nc} = \Delta KE$$

If there are no non-conservative forces, $W_{nc} = 0$, then we have

$$-\Delta PE = \Delta KE$$

$$-(PE_f - PE_i) = KE_f - KE_i$$

where the subscripts i and f mean initial and final.

$$KE_i + PE_i = KE_f + PE_f$$

$$E_i = E_f$$

where E is the total mechanical energy.

$$E \equiv KE + PE$$

$E_i = E_f$ is the statement of Conservation of Energy. Whenever there are no non-conservative forces, the total mechanical energy stays the same. You can use any labels for the points, it does not have to be the "beginning" and the "end." $E_i = E_f = E_1 = E_2 = E_a = E_b$.

I find it convenient to label the two points, i and f, or 1 and 2, or whatever you want to call them. Then use those points in $E_i = E_f$.

Springs! On Monday, we just had time to mention that the spring force is

$$F_s = -kx$$

where x is the displacement from equilibrium.

6. A spring with constant $k = 10N/m$ has an unstretched length (equilibrium length) of 14cm. When you hang a 500g mass from the spring vertically, how much does it stretch when it comes to rest again?

7. Starting from the definition of potential energy,

$$\Delta PE = -W$$

and using the best definition of work,

$$W = \int \vec{F} \cdot d\vec{r}$$

show that for a spring,

$$PE_s = \frac{1}{2}kx^2$$

8. A 2kg mass is connected to a horizontal spring with spring constant of 15N/m. The mass is pulled 25cm from equilibrium and released. How fast it is going when it passed through the equilibrium position?

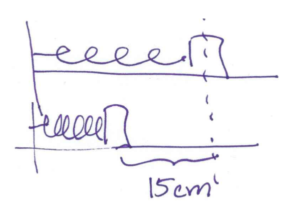
9. A 2kg mass traveling at a speed of 5m/s across a horizontal, frictionless surface, hits a spring with constant $k = 150N/m$. How much will the spring be compressed?

10. A spring of constant 25N/m is compressed horizontally a distance of 15cm from equilibrium. A mass of 200g is placed at the end of the spring and the spring is released.

    (a) What is the potential energy of the spring when it is compressed 15cm?

    (b) If the surface between the table and the mass is frictionless, how fast will the block be traveling when it leaves the spring? How far will it go?

    (c) Friction is a non-conservative force. If there is friction, you cannot use $E_i = E_f$. You must go back to $W_c + W_{nc} = \Delta KE$. From there, show that when there is friction (or other non-conservative forces), we can use:

    $$E_i + W_{nc} = E_f$$

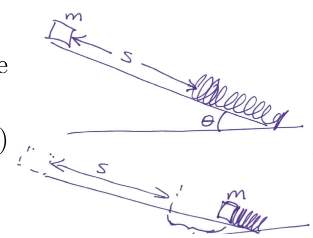
    (d) If the coefficient of kinetic friction between the block and the table is 0.3, how fast will the mass be traveling when it leaves the spring?

    (e) In the case with friction, how far will the mass travel before coming to rest?

11. An object of mass 1.5 kg is released from rest so that it slides a distance $s = 1.25m$ down a smooth incline that makes an angle of $30°$ with respect to the horizontal. The mass will hit a spring of constant k and compress it some distance $d = 20cm$ before coming to rest.

    (a) What is the speed of the block just before it hits the spring?

    (b) By how much will the spring compress? (What is k?) Save your answer!



12. Repeat the last problem with friction. Assume the coefficient of kinetic friction between the block and the incline is 0.2. Assume the spring stays the same–that means the distances cannot (both) be the same. Let's solve for s to keep the same d and k.