

**11/15 In Class – Fluids and SHM**

Fluids: Summary

$$\rho = \frac{m}{V}$$

$$P = \frac{F}{A}$$

$$\Delta P = \rho g \Delta h$$

1. A 10 cm cube of water sits on the table. How much pressure does it exert on the table?
2. We discuss two units of pressure in lecture: Pascals ( $1 \text{ Pa} = 1 \text{ N/m}^2$ ) and atmospheres ( $1 \text{ atm} = 101 \text{ kPa}$ ). There is another: mm Hg, which stands for mm of Mercury. How high of a column of mercury would it take to equal one atm?

Gauge pressure

Many devices measure gauge pressure, that is, the pressure relative to atmospheric pressure.

$$P = P_0 + \rho g \Delta h$$

where  $P$  is the absolute pressure,  $P_0$  is atmospheric pressure, and  $\rho g \Delta h$  is the pressure at some depth below atmospheric pressure.

3. Calculate the pressure difference between the surface of the water and 3m deep.
4. The absolute pressure 3m under the water is the pressure you just calculated plus atmospheric pressure. Calculate this. Explain why this new calculation is the absolute pressure.

### SHM: Summary

Simple Harmonic Motion (SHM) is defined as motion where the force or acceleration is proportional to the negative of the displacement.

$$F_x \propto -x \quad a_x \propto -x$$

The constant that makes the proportionality an equality is  $m\omega^2$  for force, or  $\omega^2$  for acceleration.

$$F_x = -m\omega^2x \quad a_x = -\omega^2x$$

where  $\omega$  is the angular velocity which is the same as the angular frequency. Recall that for a spring,  $\omega = \sqrt{\frac{k}{m}}$ , and for a simple pendulum,  $\omega = \sqrt{\frac{g}{l}}$

We determined that you can calculate the total energy of the SHO in two ways:

$$E = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$$

5. An object moves in SHM with its equation of motion given as:  $x = .2 \sin(\frac{\pi}{2}t)$
- (a) What is the amplitude of the motion?
  - (b) What is the period of the motion?
  - (c) Where is the object at  $t = 0s$ ?
  - (d) Where is the object at  $t = 1s$ ?
  - (e) Where is the object at  $t = 2s$ ?
  - (f) Differentiate this equation to get the velocity of the object as a function of time.
  - (g) What is the maximum velocity and where does it occur?
  - (h) The object has a mass of 200g. What is the maximum kinetic energy of the object? What is its total energy?

6. A mass of 1.5 kg hangs from the end of a spring of constant 200N/m
- (a) If I displace it 10cm and let it go, what is the period of its motion?
  - (b) Write the equation of motion for the mass. ( $x$  as a function of  $t$ .)
  - (c) What is the total energy of the mass? (Assuming there is no friction.)
  - (d) What happens to the motion (what is different, what is the same) if I pull it back twice as far and let go?
  - (e) Write the equation for the second motion as well.
7. Small angle approximation  
The simple pendulum is only SHM for small angles from the vertical. The small angle approximation we used is  $\sin \theta \approx \theta$ . This only works in radians. Try it for the following angles. Write  $\theta$ ,  $\sin \theta$ , and the percent difference (divide by  $\theta$ ) for:
- (a)  $\theta = 0.05$  rad
  - (b)  $\theta = 0.1$  rad
  - (c)  $\theta = 0.2$  rad
  - (d)  $\theta = 1.0$  rad
  - (e) What angle gives a 10% difference? Write that angle in degrees. Comment on what you think is a small angle.
8. A simple pendulum is made with a 200g mass hanging from a string of length 1.5 m. If the mass is displaced a small angle from equilibrium, what will be the period of its motion?