9/11 In Class Group Work–1D Kinematics

Let's try an experiment today. I'd like for you to divide into small groups and work at the white boards. Try to spread out into groups of three or four. Introduce yourselves!

- Write your names on the board so I can figure out who's in which group. I'll also take pics of the names to give you all participation points.
- Please also leave your work on the board until I see it! I will also take pics of the solutions on the white boards and post them to a google drive folder that I'll share with you all.
- Work each problem in order, all together. Some parts of this go in a progression – it matters that you do them in order. (In other words, one person in the group should not be working on problem one while someone else is working problem two, etc.)
- I will come around the room and help each group. It will take me a bit of time–feel free to talk to other groups if you get stuck!

Special case: constant acceleration,

Last time, I started from constant acceleration, $a(t) = a_0$, and the definition of acceleration:

$$a \equiv \frac{dv}{dt}$$

and the inverse operation

$$v = \int a dt$$

and for the case of constant acceleration, I used

$$v = \int a_0 dt$$

and I found that

$$v = v_0 + a_0 t$$

Thiis is a kinematic equation.

See if you can use the equation. Try these examples all together in your group. Work on the white board where possible!

1. I'm in lab measuring the motion of a cart on a track. I start my stopwatch when it is moving 1.5 m/s to the right. The cart accelerates at a constant rate of 0.25 m/s^2 to the right. What is the cart's velocity when my stopwatch reads 3.00s?
2. I'm in lab measuring the motion of a cart on a track. I start my stopwatch when it is moving 1.5 m/s to the right. The cart accelerates at a constant rate of 0.25 m/s^2 to the *left*. What is the cart's velocity when my stopwatch reads 3.00s?
3. You hear car ads that claim the car can go from 0 to 60 (mph) in 2.7 seconds. What is the car's acceleration?

There's one more kinematic equation we need: the one for position. I can repeat the same process I used above. This time I use the integral of velocity to get position. (We learned that velocity is the derivative of position; that means position is the anti-derivative of velocity. Another word for anti-derivative is integral.)

(You can find an algebra-based derivation of this equation on p28 of the textbook.)

$$x = \int v dt$$

$$x = \int (v_0 + a_0 t) dt$$

Remember that for polynomials, I raise each power of t by one and divide by the new power, so t becomes $t^2/2$ and a constant gains one power of $t/1$. So we get:

$$x = v_0 t + \frac{1}{2} a_0 t^2 + C$$

where C is a constant. I can repeat the same trick I used in the velocity equation, plugging in $t = 0$ to get

$$x_0 = 0 + 0 + C$$

and I realize $C = x_0$.

So the kinematic equation for position is:

$$x = x_0 + v_0 t + \frac{1}{2} a_0 t^2$$

We now have the two kinematic equations! They can be used any time there is a constant acceleration. (Some books derive a third and fourth, but they are pure algebra, solving one for t and plugging into the other.)

The Kinematic Equations

$$v = v_0 + a_0 t$$

$$x = x_0 + v_0 t + \frac{1}{2} a_0 t^2$$

Let's do some examples with the position equation:

4. I'm in lab measuring the motion of a cart on a track. I start my stopwatch when the cart is 10cm to the right of my origin, and it is moving 1.5 m/s to the right. The cart accelerates at a constant rate of 0.25 m/s^2 to the right. What is the cart's position when my stopwatch reads 3.00s?

5. I'm in lab measuring the motion of a cart on a track. I start my stopwatch when the cart is 10cm to the right of my origin, and it is moving 1.5 m/s to the right. The cart accelerates at a constant rate of 0.25 m/s^2 to the left. What is the cart's position when my stopwatch reads 3.00s?

6. I'm in lab measuring the motion of a cart on a track. I start my stopwatch when the cart is 10cm to the right of my origin, and it is moving 1.5 m/s to the left. The cart accelerates at a constant rate of 0.25 m/s^2 to the right. What is the cart's position when my stopwatch reads 3.00s?

It is fairly common to need both equations to answer one problem! Let's try one of those.

7. I drop an eraser from a height of 1.5 m above the floor. ('Drop' means that I let it go with initial velocity of zero. I'd have to throw it to impart an initial velocity.) The eraser accelerates at a constant 10m/s^2 down.
   (a) Use the position equation to find how long it takes to hit the ground. How long does it take?
   (b) Now you have the time it takes. Use the velocity equation to find the velocity of the eraser just before it hits the ground.

It turns out that **the acceleration of every object in free fall (near the surface of the earth) is 9.8 m/s^2 toward the center of the earth (down).** It turns out, this is the gravitational field near the surface of the earth. Thus, we use the symbol, $\vec{g}$ for what I will call "the acceleration due to gravity near the surface of the earth." Many of you will want to say "gravity" for short. But that's incorrect–gravity is a force, and this is a field which turns out (in this ONE case) to be the same as the acceleration. You can say it, we're all lazy, but remember that it is not a force!

Given $g = 9.8m/s^2$ (down),

8. I throw the eraser straight down with an initial speed of 2m/s. It takes 0.3 seconds to hit the ground.
   (a) What is the eraser's velocity just before it hits the ground?
   (b) From what height above the ground did I throw?

9. This time I throw the eraser straight up into the air. The eraser leaves my hand with a velocity of 3.2 m/s up.
   (a) If I throw an object up, it must come down. If I throw it straight up, what do you think is the velocity, for just one instant in time, at the very top? Can you explain your answer? (in words is fine.)
   (b) How long does it take to get to the top?
   (c) How high does it go?
   (d) If I catch it in my hand so that my hand is at the exact same height from which I threw the eraser, how long do you think the eraser was in the air? Why?

We have just done almost every variation of 1D motion with constant acceleration! If you can do these, you are in good shape for this material.

Let's try a more abstract problem.

10. In problem 7, you used the velocity equation (first kinematic equation) to solve for time. Do that again, but leave variables in your equation for t. No numbers. Now take that expression for t that you just found (in terms of v, v_0 and a_0, and plug it into the position equation (second kinematic equation). Rearrange everything so that you have $v^2 =$. What do you get? Check with me. Some books call this another kinematic equation. But as you can see, it is not necessary. You can always do the two steps separately.

11. Now that you have this third equation, you can do problems like 7b all in one step (without solving for 7a, the time, first). Try it. Try 7b using the new v^2 equation.