10/11 In Class–Work, KE, PE, and Conservative Forces

Summary from last time

$$W \equiv \int \vec{F} \cdot d\vec{r}$$

For constant forces, this becomes:

$$F \Delta r \cos\theta = F_x \Delta x + F_y \Delta y = F_{//} \Delta r$$

Energy $\equiv$ ability to do work. Both work and energy are measured in Joules. Both are scalars, meaning they have no direction. (They can be positive or negative, and that matters.)

Today we will learn two kinds of energy: Kinetic Energy (KE) is the ability to do work because of motion. Potential Energy (PE) is the ability to do work because of position/location.

Kinetic Energy is defined:

$$KE \equiv \frac{1}{2} m v^2$$

1. A baseball player can pitch a baseball of mass 150 g at speeds of 40 m/s. Find the kinetic energy of this baseball. Show you calculation of the units as well.

There is an important theorem called the Work–Kinetic Energy Theorem. I'll derive it here for you using calculus, and your book has a derivation (section 6.3) using algebra.

We start with the total work done on an object, which I'll call W_{net}, for short.

$$W_{net} = \int \vec{F}_{net} \cdot d\vec{r}$$

I can use Newton's Second Law, $\vec{F}_{net} = \Sigma \vec{F} = m\vec{a}$, and substitute $m\vec{a}$ into this equation.

$$W_{net} = \int m\vec{a} \cdot d\vec{r}$$

And I can rewrite $\vec{a}$ as $\frac{d\vec{v}}{dt}$, and I get:

$$Wnet = \int m\frac{d\vec{v}}{dt} \cdot d\vec{r}$$

And I can rearrange, like so:

$$Wnet = \int m d\vec{v} \cdot \frac{d\vec{r}}{dt}$$

But $\frac{d\vec{r}}{dt}$ is just $\vec{v}$!
And so I get:

$$Wnet = \int m d\vec{v} \cdot \vec{v}$$

Both $\vec{v}$ and $d\vec{v}$ are in the same ($\vec{v}$-) direction, and so it becomes:

$$Wnet = \int mvdv \cos 0 = \int mvdv$$

So I will integrate over v from v_i to v_f, and this looks like:

$$Wnet = \int_{v_i}^{v_f} mvdv$$

I integrate and get:

$$Wnet = \frac{m}{2}v^2|_{v_i}^{v_f}$$

which is:

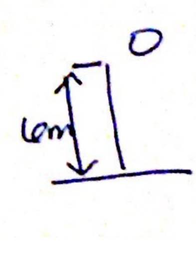
$$Wnet = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

which is **the Work – Kinetic Energy Theorem**:

$$Wnet = \Delta KE$$

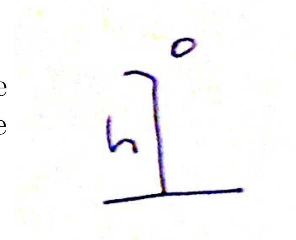
2. A 1.5 kg ball is dropped from a height of 6m.

   (a) How much work does gravity do on the ball ?
   (b) Use the Work-KE Theorem to find the speed just before it hits the ground.
   (c) Check your answer using kinematics.



3. (Same as last example, but with symbols, rather than numbers.)

A ball of mass m is dropped from a height of h above the ground. How much work does gravity do on the ball (in terms of m, g, and h)?

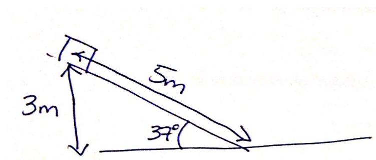


4. A block of mass 4kg slides down a smooth (frictionless) incline. The incline makes an angle of 37° wrt the horizontal. The block slides a distance of 5m down the incline, over a vertical height of 3m. (Notice that $\sin 37° = 3/5 = \cos 53°$.)

   (a) How much work does gravity do on the block as it slides?

   (b) How much work does the normal force do on the block? (For a frictionless surface, those are the only two forces acting.)

   (c) Use the Work-KE Theorem to find the speed just before it hits the ground.

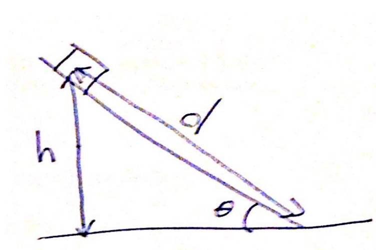


5. A block of mass m slides down a smooth (frictionless) incline. The incline makes an angle of θ wrt the horizontal. The block slides a distance of d down the incline, over a vertical height of h. (Notice that $\sin\theta = h/d = \cos\phi$ where $\phi = 90 - \theta$.)

   (a) How much work does gravity do on the block as it slides?

   (b) How much work does the normal force do on the block? (For a frictionless surface, those are the only two forces acting.)

   (c) Use the Work-KE Theorem (3rd equation above) to find the speed just before it hits the ground.



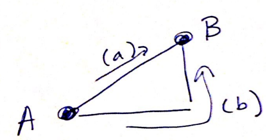
Conservative forces

Notice that the work done by gravity is the same in problems 3 and 5a. That is not a coincidence. Gravity is a conservative force. Conservative forces are those for which the work done is independent of the path. The only thing that mattered is the part of the distance parallel to the force. In this case, since gravity is down, the vertical component of the displacement was all that mattered.

All the other forces we have studied so far: friction and tension are non-conservative.

6. Consider a rough horizontal table.

   Imagine sliding a 1.5 kg book along the surface of the table from point A to point B along the two paths shown. The coefficient of kinetic friction between the book and table is 0.2. Path (a) is along the hypotenuse of a 3-4-5 (meter) right triangle. Path (b) is along the 4m side then the 3m side to get to the same end point. Find the work done by friction in each case. It should NOT be the same.

Potential Energy

For every conservative force, we can define a potential energy,

Potential Energy, PE

PE $\equiv$ the ability to do work because of location or position.

PE $\equiv$ the work you must do to move the object into position

PE $\equiv$ the negative of the work the force does as the object is moved into position.

7. Can you explain why the three definitions above are equivalent? Hint: Consider these two questions: if you lift object of mass m up a distance h, how much work do you do? How much work does gravity do?

   The last one is the easiest to write as an equation.

$$\Delta PE_f \equiv -W_f$$

   I read the above equation as: the change in potential energy associated with a (conservative) force is equal to the negative work done by that force.

You have already done this for gravity (near the surface of the earth.)

$$PE_g = mgh$$

Conservation of Energy

Starting from the Work-KE Theorem:

$$W_{net} = \Delta KE$$

The total work can be split into the work done by conservative forces W_c and the work done by non-conservative forces, W_{nc}. Then we have

$$W_c + W_{nc} = \Delta KE$$

If there are no non-conservative forces, $W_{nc} = 0$, then we have

$$-\Delta PE = \Delta KE$$

$$-(PE_f - PE_i) = KE_f - KE_i$$

where the subscripts i and f mean initial and final.

$$KE_i + PE_i = KE_f + PE_f$$

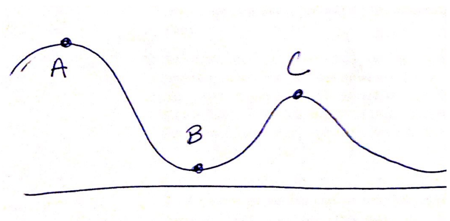
$$E_i = E_f$$

where E is the total mechanical energy.

$$E = KE + PE$$

$E_i = E_f$ is the statement of Conservation of Energy. Whenever there are no non-conservative forces, the total mechanical energy stays the same.

8. A famous conservation of energy problem is the roller coaster. Only the height matters for the potential energy, so you can find the speed at any point if you know the speed at any one height. For the picture shown here, A is at a height of 20m, B is at 3m, and C is at 10m. If the roller coaster has a speed of 2m/s at A, what is its speed at B and C?



9. A simple pendulum consists of a mass on the end of a string.

A mass of 1.5 kg swings on the end of a simple pendulum of length 40cm The pendulum bob is released from rest when the string makes an angle of 30° wrt the vertical. Find the speed of the bob as it passes through the center point at the bottom of the arc.