§ 1 Introduction

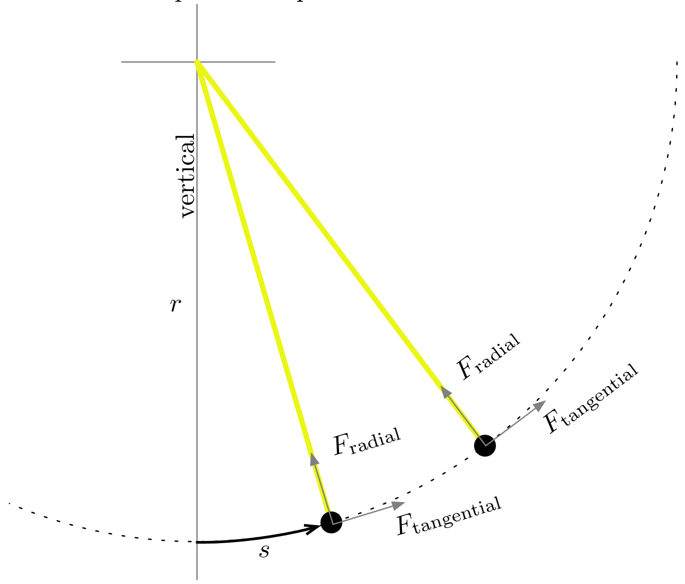
In this laboratory you will investigate Newton's Second Law. Newtons second law states

$$\Sigma F = ma$$

where a is the acceleration of an object, m is the mass of the object and ΣF is the sum of all forces on the object.

§ 2 Background

We will look at the motion of a simple pendulum, a mass swinging back and forth on the end of a string. While the motion appears to be two dimensional, because of the string the mass is constrained to move along the arc of a circle, so it can be thought of as a one dimensional problem. We will use the coordinate s to describe the position of the mass along the circle. The coordinate s is the location along the circle as measured along the arc from the equilibrium position at the bottom.



Position $s = 0$ is when the pendulum is hanging straight down, positive s is when the pendulum is on the right and negative s is when the pendulum is on the left.

In this case we can say that the acceleration along the arc is

$$a = \frac{d^2 s}{dt^2}$$

and Newton's Second Law is

$$\Sigma F = ma = m\frac{d^2 s}{dt^2}$$

where ΣF is the net force on the object along the arc. We call this the tangential force, because it is a tangent to the circle.

Notice that the sting of the pendulum does not apply any force in the tangential direction because a radius of the circle is perpendicular to a tangent of the circle. So we need not consider the force of the string when we evaluate ΣF. From here on we will just think about motion and forces along the arc of the circle.

§ 3 Setup a pendulum and record

Set up a pendulum as shown in these videos step 1, step 2, and step 3.

As explained in the step 3, record a video of the pendulum as you lift the pendulum, stopping the positions where reading on the scale is -0g, -10g, -20g, -30g, -40g, -50g, -60g, -70g, -80g, and -90g. Be especially careful with the -90g position as the string going down to the scale can get caught up on the lower part of the pendulum mass, causing a misreading of the force.

Now disconnect the second string that was used to measure the force, and then record the pendulum swinging back and forth. Start the pendulum moving by pulling the pendulum up very high, with the string nearly horizontal, then letting go. Record the motion for at least three full swing cycles. The pendulum will not always swing in the plane that you want (parallel to the poster board), so keep starting over until the oscillation is in that plane. In the case that your camera has the ability to record video at different frame rates, set the frame rate to 60 fames per second. More frames per second is ok too since you can skip frames in Tracker, or just do more clicking. It is ok to use 30 frames per second too if that is the only option, but the measurements of acceleration will have more noise.

Measure and record the mass of the pendulum.

§ 4 Determine force as a function of position

Now you will use the first video that you recorded to evaluate how the force changes as the position of the pendulum changes.

Take a quick look at your pendulum. Pull the pendulum a little to the left. Notice that there is a force that pulls to the right, toward to the center. Now displace it to the right. Notice that there is a force that pulls it back to the left, toward the center. We will call this the force F_c, the force that is pulls the pendulum toward the center.

Now consider what happened when we connected the string that lead to the scale. The string is under tension and so applies a force on the pendulum. The same tension is pulling up on the scale, so the force applied to the pendulum F_{string} is same as the force measured by the scale. The scale reads in grams because it is assuming that you have place an object on the scale so the scale measures the weight (mg) and divides by g in order to give the reading on the screen. Thus we need to multiply the reading on the screen (m_{screen}) by g in order to get the force.

$$F_{\text{string}} = m_{\text{scale}} g$$

(The string was pulling up on the scale so the mass reading on the scale was negative but we will ignore this negative sign in the calculation above since we have already taken into account the direction of the force.)

When we held the pulley still, the pendulum was not moving so that the acceleration of the pendulum was zero. This means that the net force was zero too, since $\Sigma F = ma = 0$. But the only forces are F_c and F_{string} so $\Sigma F = F_c + F_{\text{string}}$ and

$$0 = \Sigma F = F_c + F_{\text{sting}} \longrightarrow F_c = -F_{\text{string}}$$

so

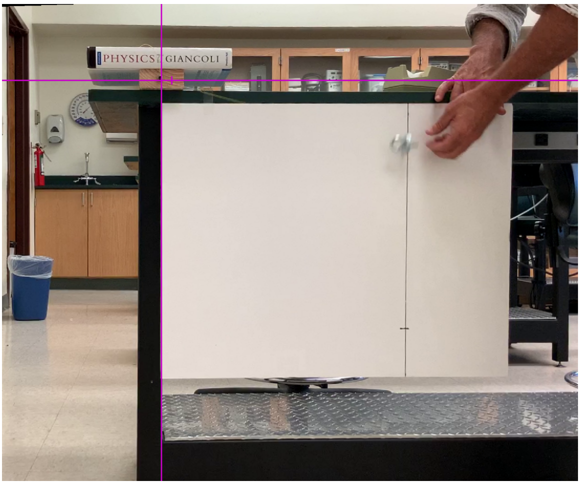
$$F_c = -m_{\text{scale}} g$$

Thus the force applied by the string connected to the scale is equal and opposite to F_c. So the reading on the scale also gives us the magnitude of the force that is trying to pull the mass back to the center! In this way we are able to measure the force F_c as a function of the position s.

When the string is disconnected the force F_c is the **only** force applied and the pendulum accelerates toward the center.

Procedure 4.a

Take your first video into Tracker, correct the perspective using the poster board as the reference rectangle if necessary and set the scale. Set the axes so that the origin is at the point from which the pendulum swings (the center of the arc). It is very important for this lab that the vertical axis in tracker is parallel to the edge of the poster board, as shown in the image below. In the image the axis is not only parallel to the edge but also coincident, it only needs to be parallel.



Now begin tracking the pendulum. Use the center of the top washer as the point on the mass to track. First track ten consecutive frames of the position of the pendulum when it is hanging straight down. Next skip forward without tracking any frames until you get to the position of the pendulum where the scale reads 10 grams, track at least ten consecutive frames of this position. Now skip forward without tracking any frames until you get to the position of the pendulum where the scale reads 20 grams, track at least ten consecutive frames. Repeat for 30g, 40g, 50g, 60g, 70g, 80g and 90g. Save the table of data to a file called `force.txt`.

Get the applications for this lab [MSwin, macOS] from the class website. Run `Lab2p1` and have the app load the data file `force.txt` and input the mass of the pendulum in grams. The program will read the tracked data and determine the average position s for each of the 10 positions (0g to 90g). Next the program graphs the force $F_c = -m_{\text{scale}}g$ versus the position s. The program will also graph a model force versus position curve. You have the choice of either a sinusoidal function, a quadratic or a linear function. Try all three options. A pdf of each graph will be saved.

▷ QUESTION 1
The theoretical prediction for the force is the sinusoid $F_c = -mg\sin(s/r)$. Is this function the best match to your measurements?

§ 5 Does $a = \frac{F_c}{m}$?

Now take your second video into Tracker. As you did with the previous video, correct the perspective, and set the origin, axis, and scale. Track at least two full cycles of the oscillation. Save the data to the file `swing.txt`.

Launch the app `Lab2p2` and load the data file `swing.txt`.

The app computes the position s for each tracked frame and graphs s versus t. From this the acceleration $a = \frac{d^2s}{dt^2}$ is computed and graphed. You can choose the number of points to use in computing the acceleration with the pull down option box.

In the first part of this lab we verified that the force pulling the pendulum back to the center is $F_c = -mg\sin(s/r)$. So we know F_c as a function of the position s. In addition with Tracker we have measured the position s at a sequence of times t. Thus for each time we can compute the force from the measured position at that time. In this way $\frac{F_c}{m}$ is computed for each time and graphed along with the acceleration.

▷ QUESTION 2
Do the graph of $a = \frac{d^2s}{dt^2}$ and $\frac{F_c}{m}$ look the same? Would you say that they are equal? Does Newton's Second Law apply to your measured motion?