§ 1 Introduction

In this laboratory we will observe the momentum of objects in a collision. With any luck we will observe one of the most fundamental conservation laws of nature. The momentum of an object with mass m and velocity $\vec{v}$ is defined to be

$$\vec{p} = m\vec{v}$$

§ 2 Record a video and track it

Procedure 2.a

First record a video of some collisions.

1) Find the white disks in the DIY lab kit. There should be three, one that is thicker than the other two.
2) Find a smooth horizontal surface, ideally the surface would be so smooth that it is slippery. A desk or kitchen counter are good candidates. The floor might be good too, but not if it has grooves in it. You will need an area of about the size of your poster board. Mark the corners of a reference rectangle on the surface with visible tape. Use the poster board to be sure that the sides of the rectangle are square to each other.
3) Set up your video camera so that you can see the full rectangle in the video.
4) Set your camera to 60 frames per second if it has that capacity.
5) Place one of the thinner disks in the center of the area. Practice sliding the thick disk so that it strikes the thin disk, and they both slide away in different directions after the collision. Try to avoid the head on collisions where they go the same direction after. Try to get the speed so that the disks going off slide at least 20 cm before stopping.
6) Set yourself up so that you do not interferer with the view of the camera when you are sliding the disk.
7) Once you have figured out your sliding technique, you can make a video of some collisions. Get at least two good ones. Then do the same but with using the two thin disk, instead of the thick disk and the thin disk, again recording at least two good collisions.

Procedure 2.b

Next use Tracker to track the motion of the good collisions. We will refer to the disks as the incident disk and the target disk. The target disk is the one that was hit, and the incident disk is the one that you slid into the target disk.

1) Use the perspective filter and your reference rectangle to correct the perspective.
2) Set the scale. Don't worry about the axes, they can be in any orientation.
3) Find the first good collision in the video.
4) Track the motion of the incident disk before the collision. It is critical that you **stop** tracking after tracking the last frame before the collision, no sooner and no later. You should track about ten consecutive points before the collision. Save this data to a file `1Ii.txt`.
5) Track the motion of the incident disk after the collision. It is critical that the first point tracked is the first point after the collision. Track about ten consecutive points after the collision. Save the data to a file called `1If.txt`
6) Track the motion of the target disk after the collision, as you did the incident disk after the collision. Save the data to a file called `1Tf.txt`.
7) Repeat the above for the next collision using the names `2Ii.txt`, `2If.txt` and `2Tf.txt`.
8) Repeat for the two collisions of a thin target disk with a thin incident disk, numbering the files for these collisions 3 and 4.

§ 3 Analyzing the tracks to determine the velocity

Procedure 3.a

1) Get the application for this lab [macOS, [MSwin] from the class website. Run `Lab4p1`, and feed it the file `1Ii.txt`. Set the pulldown menu to "`before` collision". The program will plot the coordinates x and y versus time and estimate the velocities v_x and v_y just before the collision. It will also save the graph as a PDF.
2) Do the same with the the data for the two outgoing disks `1If.txt` and `1Tf.txt`, but with these files set the pull down menu to "`after` collision". This will cause the program to estimate the velocity at the beginning of the track rather than the end.
3) On a piece of graph paper plot the vectors for the initial total momentum

$$\vec{p^i} = \vec{p^i_I} + \vec{p^i_T} = \vec{p^i_I} + 0$$

and final total momentum

$$\vec{p^f} = \vec{p^f_I} + \vec{p^f_T}$$

and the change in momentum

$$\vec{\Delta p} = \vec{p^f} - \vec{p^i}.$$

An example of making the graph and the associated calculations is on the last page of this document.

▷ Question 1

Measure the length of $\vec{\Delta p}$ with a ruler to determine its length in $\frac{\text{g}\cdot\text{cm}}{\text{s}}$. Based on the length of $\vec{\Delta p}$ does this data appear to confirm or refute that the total initial momentum is the same as the total final momentum? When making this evaluation you should keep in mind that we have an uncertainty in our estimates of the momentum. This uncertainty will be estimated in the following paragraph.

In the first lab this semester we estimated the measurement uncertainty in the velocity, and found that it was arounds $2\frac{\text{cm}}{\text{s}}$. Because we used ten points to estimate the velocity we should have a smaller uncertainty, let's estimate about $0.5\frac{\text{cm}}{\text{s}}$. So for a mass of 6 grams we expect a measurement uncertainty in the momentum of about $\delta p = 3\ \frac{\text{g}\cdot\text{cm}}{\text{s}}$, and $4.5\ \frac{\text{g}\cdot\text{cm}}{\text{s}}$ for a 9 gram mass. But in computing the change in momentum we used three independently measured velocities

$$\vec{\Delta p} = \vec{p^f} - \vec{p^i} = \vec{p^f_I} + \vec{p^f_T} - \vec{p^i_I}$$

so the uncertainties add

$$\delta[\vec{\Delta p}] = \delta[\vec{p}^f_I] + \delta[\vec{p}^f_T] + \delta[\vec{p}^i_I] = 3\ \frac{\text{g}\cdot\text{cm}}{\text{s}} + 4.5\ \frac{\text{g}\cdot\text{cm}}{\text{s}} + 4.5\ \frac{\text{g}\cdot\text{cm}}{\text{s}} = 12\ \frac{\text{g}\cdot\text{cm}}{\text{s}}$$

Thus when you evaluate if the initial and final momentum are the same or not we must evaluate the change in momentum compared to this uncertainty.

▷ Question 2

Compute the initial and final kinetic energies and their uncertainties.

$$K_i = \tfrac{1}{2} m_I (v^i_I)^2$$

$$K_f = \tfrac{1}{2} m_I (v^f_I)^2 + \tfrac{1}{2} m_I (v^f_T)^2$$

Do these appear to be the same.

▷ Question 3

Answer questions 1 and 2 for the other three collisions. Make general conclusions based on what happened in the four collisions. Was Δp never, sometimes or always zero? Was ΔK never, sometimes or always zero?

(example calculation on next page)

Example answer to question 1

An example of the calculation and plotting of the vectors is shown below, were the velocities estimated by `Lab4p1` are the left most column, m_I is the mass of the incident disk and m_T is the mass of the target disk.

$$m_I = 6.0\text{g} \quad \text{and} \quad m_T = 6.0\text{g}$$

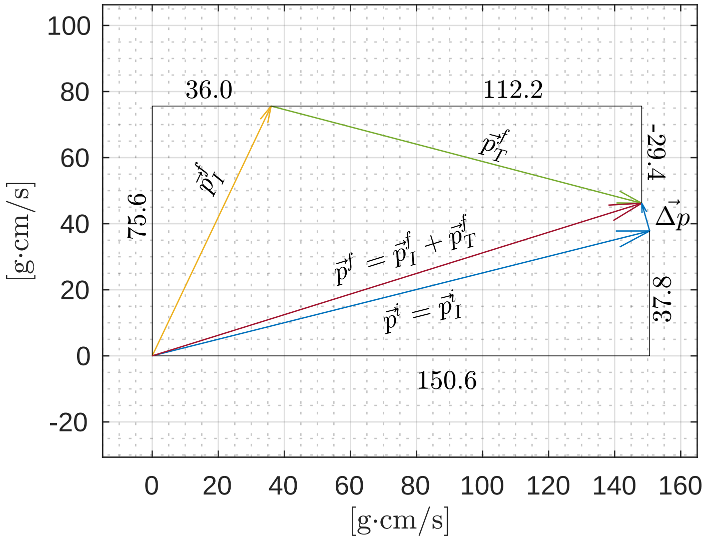
$$\left.\begin{array}{l} v^i_{I\,x} = 25.1\dfrac{\text{cm}}{\text{s}} \longrightarrow p^i_{I\,x} = m_I v^i_{I\,x} = 150.6\ \dfrac{\text{g}\cdot\text{cm}}{\text{s}} \\ v^i_{I\,y} = 6.3\dfrac{\text{cm}}{\text{s}} \longrightarrow p^i_{I\,y} = m_I v^i_{I\,y} = 37.8\ \dfrac{\text{g}\cdot\text{cm}}{\text{s}} \end{array}\right\} \vec{p^i_I}$$

$$\left.\begin{array}{l} v^f_{I\,x} = 6.0\dfrac{\text{cm}}{\text{s}} \longrightarrow p^f_{I\,x} = m_I v^f_{I\,x} = 36.0\ \dfrac{\text{g}\cdot\text{cm}}{\text{s}} \\ v^f_{I\,y} = 12.6\dfrac{\text{cm}}{\text{s}} \longrightarrow p^f_{I\,y} = m_I v^f_{I\,y} = 75.6\ \dfrac{\text{g}\cdot\text{cm}}{\text{s}} \end{array}\right\} \vec{p^f_I}$$

$$\left.\begin{array}{l} v^f_{T\,x} = 18.7\dfrac{\text{cm}}{\text{s}} \longrightarrow p^f_{T\,x} = m_T v^f_{T\,x} = 112.2\ \dfrac{\text{g}\cdot\text{cm}}{\text{s}} \\ v^f_{T\,y} = -4.9\dfrac{\text{cm}}{\text{s}} \longrightarrow p^f_{T\,y} = m_T v^f_{T\,y} = -29.4\ \dfrac{\text{g}\cdot\text{cm}}{\text{s}} \end{array}\right\} \vec{p^f_T}$$

We need to pick a scale for our plot of momentums. Using the values above we see that the greatest length is 150.6 $\frac{\text{g}\cdot\text{cm}}{\text{s}}$, so we should pick the scale so that this fits on the page. So if I have graph paper that is in cm will choose my scale to be 1 cm = 10 $\frac{\text{g}\cdot\text{cm}}{\text{s}}$. If I have graph 1/4" graph paper then I would choose 1/4" = 5 $\frac{\text{g}\cdot\text{cm}}{\text{s}}$. These scales might be different for your collision.

These computed components of momentum lead to the following graph. Notice that we need to do the "head-to-tail" vector addition of $\vec{p^f_I}$ and $\vec{p^f_T}$ in order to find the final momentum $\vec{p^f} = \vec{p^f_I} + \vec{p^f_T}$.



Using the ruler to measure the length of $\vec{\Delta p}$ we find about 9 $\frac{\text{g}\cdot\text{cm}}{\text{s}}$. This is less than our uncertainty so this data does not show a clear change in momentum. This data is consistent with $\vec{\Delta p} = 0$

continued on next page

Example answer to question 2

The kinetic energy of a particle is

$$K = \tfrac{1}{2}mv^2 = \frac{p^2}{2m}$$

which can be compute from the ruler measured length of the momentum vector in the graph. The uncertainty in its kinetic energy is

$$\delta K = \frac{dK}{dv}\delta v = mv\ \delta v = p\ \delta v = p\ (0.5\frac{\text{cm}}{\text{s}})$$

which can also be computed from the ruler measured length of the momentum in the graph. So we can compute

$$\begin{aligned} K^i &= 200.9 \pm 7.8\mu\text{J} \\ K_I^f &= 58.4 \pm 4.2\mu\text{J} \\ K_T^f &= 112.1 \pm 5.8\mu\text{J} \\ K^f = K_I^f + K_T^f &= 170.5 \pm 10.0\mu\text{J} \\ \Delta K = K^f - K_i &= -30.4 \pm 17.8\mu\text{J} \end{aligned}$$

Since we have been using grams and cm for units our units for energy will be

$$\frac{\text{g}\cdot\text{cm}}{\text{s}}\cdot\frac{\text{cm}}{\text{s}} = 10^{-7}\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} = 10^{-7}\text{J} = 0.1\mu\text{J}.$$

this was used in the above computation of energies.

So we find that since ΔK is at most

$$-30.4 - 17.8 < \Delta K < -30.4 + 17.8$$
$$-48.2\mu\text{J} < \Delta K < -12.6\mu\text{J}$$

We see that that kinetic energy was lost in the collision and

$$\Delta K \neq 0.$$