HW 9/8

1.14) $\Delta t = t_2 - t_1$

$\Delta x = x_2 - x_1$

Let $I = \begin{bmatrix} c\Delta t \\ \Delta x \end{bmatrix}$, Then $I = e_2 - e_1$.

Similarly, $I' = e_2' - e_1'$ where $e' = Le$, $L = \begin{bmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{bmatrix}$

the Lorentz transformation.

$$I' = e_2' - e_1'$$
$$= Le_2 - Le_1$$
$$= L(e_2 - e_1)$$
$$= LI$$

So the space-time intervals transform via the Lorentz transformation.

1.16)

C before D - timelike
C before A - timelike
B before A - timelike
D left of A - spacelike
D left of B - spacelike
C left of B - spacelike

1.42) S: rest frame of plane
S': rest frame of barn
e_1: event of the plane's tail entering barn
e_2: event of the plane's nose exiting barn

$\Delta x = 10\,m$ $\quad\quad$ $\Delta x' = 8\,m$

$$\begin{bmatrix} c\Delta t' \\ \Delta x' \end{bmatrix} = \begin{bmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{bmatrix} \begin{bmatrix} c\Delta t \\ \Delta x \end{bmatrix}$$

$$\begin{bmatrix} c\Delta t' \\ 8\,m \end{bmatrix} = \begin{bmatrix} 2 & \sqrt{3} \\ \sqrt{3} & 2 \end{bmatrix} \begin{bmatrix} c\Delta t \\ 10\,m \end{bmatrix} \longrightarrow \begin{aligned} c\Delta t' &= 2c\Delta t + \sqrt{3}(10\,m) \\ 8\,m &= \sqrt{3}\, c\Delta t + 2(10\,m) \end{aligned}$$

$$\hookrightarrow \boxed{\begin{aligned} c\Delta t &= -6.93\,m \\ c\Delta t' &= 3.46\,m \end{aligned}}$$

In the plane's frame S, the nose leaving the barn event happens before the tail enters. The plane does not fit in the barn.
But, in the barn's frame S', e_2 happens after e_1, so the plane does fit in the barn.
The difference in these stories is not surprising since the displacement of the two events is spacelike. There is no physical significance to the time ordering of events.

9/8

1.43) a) Since $c\Delta t = c(70ns) = 21m$ and $\Delta x = 33m$, $c\Delta t < \Delta x$, Thus the interval between the lights is spacelike, not timelike. The events cannot be causally related.

b) S: frame of ship
S': frame of station
• The origin of S is set at the light near the rear.

Event	Ship ct	ship x	Station ct'	Station x'
front flash	33	0	44	55
rear flash	0	21	35	28

(ship x: given)

• Use Lorentz transformation with $\beta = 0.8$ to find last 2 columns of the chart.

From the chart we can see $c\Delta t' = 35 - 44 = -9m$. so the rear flash happens before the front.

c) From the chart we can see $\Delta x' = 28 - 55 = -27m$ so the rear flash happens first.