Exam 1 - Solutions

1/ a.

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$= \frac{1}{\sqrt{1-\left(\frac{\sqrt{3}}{2}\frac{c}{c}\right)^2}} = \frac{1}{\left(1-\frac{3}{4}\right)^{1/2}} = \frac{1}{\left(\frac{1}{4}\right)^{1/2}} = (4)^{1/2}$$

$$\gamma = 2$$

b. here, $ct = (3\cdot 10^8)\left(\frac{40}{3}\cdot 10^{-9}\right) = 4\text{m}.$

$$\begin{array}{lcc}
 & & \\
\text{leave} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
\text{front expl} & \begin{bmatrix} 4\text{m} \\ 40\text{m} \end{bmatrix} & \begin{bmatrix} \text{see below} \end{bmatrix} \\
\text{back expl} & \begin{bmatrix} 4\text{m} \\ 0 \end{bmatrix} & \begin{bmatrix} \text{see below} \end{bmatrix} \\
 & \underbrace{\qquad\qquad}_{S,\ \text{Luis}} & \underbrace{\qquad\qquad}_{S',\ \text{Josh}}
\end{array}$$

use

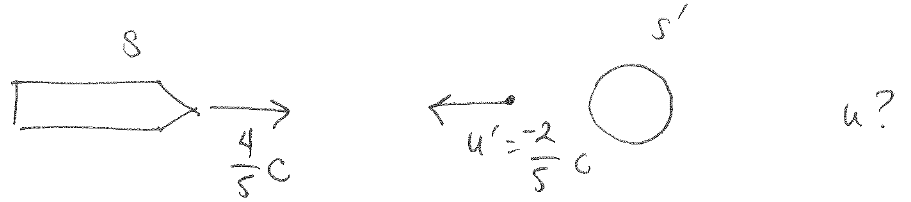
$$\underbrace{\begin{bmatrix} ct' \\ x' \end{bmatrix}}_{S'} = \begin{bmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{bmatrix} \underbrace{\begin{bmatrix} ct \\ x \end{bmatrix}}_{S}$$

front expl. $\begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} 2 & 2\cdot\frac{\sqrt{3}}{2} \\ 2\cdot\frac{\sqrt{3}}{2} & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 40 \end{bmatrix} = \begin{bmatrix} 8 + 40\sqrt{3} \\ 4\sqrt{3} + 80 \end{bmatrix}$

back expl. $\begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix} \begin{bmatrix} 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ 4\sqrt{3} \end{bmatrix}$

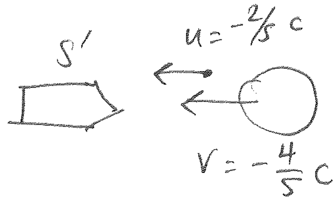
c. Ship leaves. Back explodes $\frac{8}{c}$ seconds later. An additional $\frac{40\sqrt{3}}{c}$ sec. later, the front explodes.

2/



transf. is $u' = \dfrac{u+v}{1+\frac{uv}{c^2}}$

switch reference frames.



$$u' = \frac{-\frac{2}{5}c + \left(-\frac{4}{5}c\right)}{1 + \frac{\left(\frac{2}{5}c\right)\left(-\frac{4}{5}c\right)}{c^2}} = \frac{-\frac{4}{5}c}{1+\frac{8}{25}} = \frac{-\frac{4}{5}c}{\frac{33}{25}}$$

$$= -\frac{30}{33}c = \frac{-10}{11}c$$

alternative sln. rearrange $u' = \dfrac{u+v}{1+\frac{uv}{c^2}}$ & solve for u.

$$u = \frac{u'-v}{1-\frac{u'v}{c^2}} = \frac{\left(-\frac{2}{5}c\right) - \left(\frac{4}{5}c\right)}{1 - \frac{\left(-\frac{2}{5}c\right)\left(\frac{4}{5}c\right)}{c^2}} = \text{same as above}$$

3/

energy conservation

$E_i = E_f$: $\gamma_{u_0} m_0 c^2 + \frac{hc}{\lambda} = \gamma_{u_f} m_f c^2$

momentum conservat

$p_i = p_f$: $\gamma_{u_0} m_0 u_0 - \frac{h}{\lambda} = \gamma_{u_f} m_f u_f$

4/ a. $E = \gamma_u mc^2$ and $p = \gamma_u mu$ when $\gamma_u = \dfrac{1}{\sqrt{1-\left(\frac{u}{c}\right)^2}}$

to find u...

$$\frac{p}{E} = \frac{\gamma mu}{\gamma mc^2} \longrightarrow \frac{pc^2}{E} = u$$

subs. in values

$$u = \frac{(4\epsilon)c}{12\epsilon} = \frac{c}{3}$$

b. the invariant quantities are mass (m) and rest energy (mc^2)

5/ time order of events are fixed if the interval is time like.

$$c\Delta t > \Delta x$$

here the 2 events are $\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 240 \\ 200 \end{bmatrix}$

$$ct = 3 \cdot 10^8 \cdot 800 \cdot 10^{-9} = 2400 \cdot 10^{-1} = 240$$

so $240 > 200$

- It's time-like! So order of events are always the same.
- Note that the quantity $(c\Delta t)^2 - (\Delta x)^2$ is invariant. So no matter what reference frame is used, then two events will be time like.
- The two events may be causally-related (but they don't have to be).

6/ (a) $E = hf = \frac{hc}{\lambda}$

$$= \frac{(6.63\cdot 10^{-34})(3\cdot 10^{8})}{500\cdot 10^{-9}} = 3.99\cdot 10^{-19}\,J$$

(b) $I = \frac{\text{power}}{\text{area}} = \frac{(hf)(dn/dt)}{\text{area}}$

$\frac{dn}{dt} = \frac{I\cdot \text{area}}{hf}$ $\rightsquigarrow$ where $f = \frac{c}{\lambda}$

$$= \frac{(1.5\cdot 10^{3})(1\cdot 10^{14})}{(6.63\cdot 10^{-34})\left(\frac{3\cdot 10^{8}}{500\cdot 10^{-9}}\right)} = 3.8\cdot 10^{35}\ \frac{\text{photons}}{\text{sec}}$$

7/ $\Delta E = \Delta m c^2$

$\Delta m = \frac{\Delta E}{c^2}$ here $\Delta E = \text{power}\cdot\text{time}$ $\rightarrow 1s$

$\rightarrow 1.5\cdot 10^{3}\ \frac{J}{s\ \text{area}}\cdot \text{area}$

$$= \frac{(1.5\cdot 10^{3})(1)(1\cdot 10^{14})}{(3\cdot 10^{8})^{2}}$$

$= 1.7$ kg in 1s.

8/ $\phi = hf = \frac{hc}{\lambda}$

$$\lambda = \frac{hc}{\phi} = \frac{(6.63\cdot 10^{-34})(3\cdot 10^{8})}{(1.2)(1.6\cdot 10^{-19})} = 1.0\cdot 10^{-6}\,m$$

$\rightarrow$ change eV $\rightarrow$ J

$\Rightarrow \lambda$ must be equal to, or less than, $1\cdot 10^{-6}\,m$

9/ The Lorentz transformation is $\begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$.

It will recover the Galilean transformation in the limit of $\frac{v}{c} \ll 1$.

Since $\gamma = \frac{1}{\sqrt{1-(\frac{v}{c})^2}} \to 1$ when $\frac{v}{c} \ll 1$.

$\beta = \frac{v}{c}$. Strictly speaking, $\beta \to 0$. However, let's keep its $\frac{v}{c}$ dependence, for now.

The transformation is then $\begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} 1 & \beta \\ \beta & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$

$$ct' = ct + \beta x$$

$$x' = \beta ct + x.$$

Sub in $\beta = \frac{v}{c}$, and simplify a little

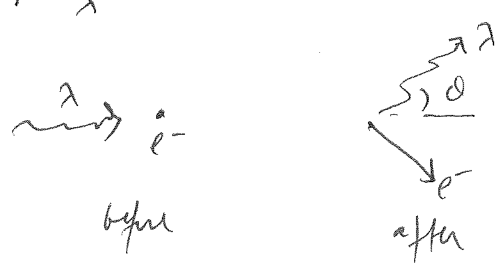
$$\boxed{\begin{aligned} t' &= t + \frac{v}{c^2} x \\ x' &= vt + x \end{aligned}}$$

$t' = t$ in limit that $\frac{v}{c} \ll 1$. However, if x is v. large, this isn't the case.

clearly Galilean.

10/ the Compton experiment is the scattering of light off free e^-, as shown below. Its results show that the scattered light has a wavelength that depends on the scattering angle, $\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$.

The results may be understood w/i the photon framework, where light has momentum $p = \frac{h}{\lambda}$ & interacts w/ the e^- like a particle.



11/ Black body radiation is the emission of light from black bodies. The spectrum of this emission has a peak, that would not be explained classically. In fact, classically, the spectrum should diverge at low wave lengths. (see picture). The problem was resolved by postulating the the energy of light is quantized, $E = nhf$, as opposed to being continuous.