1. Consider two events in the S frame, where $ct_1 = 3$ and $ct_2 = 10$.

   (a) Calculate $c\Delta t$.
   (b) Provide values of x_1 and x_2 that would lead to interval being *timelike*. Give actual numbers.
   (c) Provide values of x_1 and x_2 that would lead to interval being *spacelike*. Give actual numbers.
   (d) Provide values of x_1 and x_2 that would lead to interval being *lightlike*. Give actual numbers.

2. True or False: Two events are spacelike. The two events are therefore causally related. (That is, one can be the cause of other to happen.)

3. Finish the problem from the end of class on Friday. Calculate $p_{initial}$ and p_{final} in the second (unprimed) frame.

   Is momentum conserved?

4. (Review) Block A of mass 1.6kg moves to the right at 4m/s on a frictionless, horizontal track. Block B of mass 2.1kg moves to the left at 2.5 m/s. Blocks A and B collide. After the collision, block B is moving at 3.12 m/s to the right.

   (a) Calculate the velocity of A after the collision.

   (b) Was this an elastic or inelastic collision? Show a calculation to support your answer.

   (-3.38 m/s. elastic... you'll have to come up with the calculation)

5. Consider the relativistic form of momentum $p = \gamma_\mu m u$, where $\gamma_\mu = \frac{1}{\sqrt{1-(u/c)^2}}$.

   Show that the classical momentum is recovered in the limit that $u/c \ll 1$.

6. What does it mean for a physical property to be invariant? To answer, provide a definition of invariant and an example of a property that is invariant. (For example, is the length of an object invariant?)

   How about conserved? Provide a definition for conserved and an example of a physical property that is conserved.

7. Make bulleted list of main ideas we covered in special relativity. For each item, write a short phrase or sentence describing the idea *and* an equation. You should have 10 ± 3 items.

8. (Review) A few things to recall about electromagnetic (EM) waves from introductory physics:

   (a) Calculate the wavelengths of a
   100MHz FM radio wave
   5 GHz Wifi signal
   1 EHz x-ray

   (b) Classically, what properties determine the rate at which an EM wave transports energy? Does it depend on speed, frequency, wavelength, electric field strength, or.... ? An equation is helpful. (You can look this up. It's best to look up the intensity which has units of power/area [W/m^2].)

9. Try out the blackbody radiation app. What temperature results in the peak appearing at 450nm (blue)? 600nm (yellow)? 700nm (red)?

10. The human eye is sensitive to wavelengths of 400nm to 700nm. What is the corresponding range of photon energies? Give this in Joules.

    from Wolfson

11. Find the energy in electron volts (eV) of a
    (a) 1 MHz radio photon
    (b) 3×10^{18}Hz x-ray

    from Wolfson

12. Play around with the photoelectric effect app. You should be able to change the wavelength and the intensity (brightness, photon density).

    (a) What will increase the number of electrons ejected?
    (b) What will increase the energy of the ejected electrons?

13. (Review) A particle, of charge q and mass m, is accelerated from rest through a potential difference of $|\Delta V|$. It has a final speed of v after it's accelerated.

    (a) Derive the following expression for v

    $$v = \sqrt{\frac{2q|\Delta V|}{m}}$$

    Use expressions for kinetic energy (classical is fine), electrostatic potential energy ($U = qV$ or kq_1q_2/r... whichever is most appropriate), and energy conservation ($\Delta E = 0$).

    (b) Calculate the speed of an electron accelerated over a potential difference of $|\Delta V| = 100kV$.

14. (Review) Two slits 0.075mm apart are located 1.5m away from a screen. A green laser with wavelength 532 nm is incident on the two slits. Determine the position of the third order bright fringe (the 3rd fringe above the center of the pattern).

    You might want to review your Intro text. In Wolfson, it's chapter 32.

15. (Review) Consider two vectors: $\vec{A}$ has magnitude of 3 and points in the +y direction. $\vec{B}$ has a magnitude of 5.

    Calculate $\vec{A} \cdot \vec{B}$ when $\vec{B}$ points in the direction

    (a) +x
    (b) +y
    (c) -y
    (d) $+\pi/3$ above the +x-axis.

16. (Review) Light of some wavelength λ is incident on a diffraction grating with slit spacing of 1.67μm. The first order peak is found at angle of 17.0 degrees. The second is at 35.7 degrees. Calculate the wavelength of the light.

17. (read ahead in 4.1!) (a) Calculate the wavelength of a fighter jet (21,000 kg) flying at 300 m/s (that's a little less than the speed of sound).

    (b) Calculate the wavelength of a proton moving at 300 m/s.

18. A double slit experiment is setup with a laser having a wavelength of 650nm. You have a photodetector with a diameter of 0.1mm. The detector can be moved through the interference pattern. The average number of counts/sec at the steps are:

    30 10 0 10 30 40 30 10 0 10 30

    Determine the
    (a) intensity of the light at each step. Recall that intensity has units of W/m^2.
    (b) amplitude of the electric field at each step. Recall that $I = \frac{1}{2}c\epsilon_0 E_0^2$

    One of the slits is then closed. Determine the
    (c) amplitude of the electric field at each slit. (Hint: when *both* slits are open, the two electric fields are completely in phase at the central maximum)
    (d) number of photons that arrive at each slit

19. (Review) Consider $\vec{A} = 5\hat{x} - 6\hat{y}$ and $\vec{B} = -3\hat{x} + 2\hat{y}$. Calculate

    (a) the sum $\vec{A} + \vec{B} = \vec{C}$
    (b) the magnitude and direction of $\vec{A}$
    (c) the magnitude and direction of $\vec{B}$
    (d) Plot $\vec{A}$ and $\vec{B}$.

    Give direction as an angle with respect to the +x axis. Clockwise is a positive angle.

20. (Review) Some properties and calculations with exponential function e^x.

Let $a_1 = 6e^{3t}$, $a_2 = 3e^{-4t}$, and $a_3 = 5e^{-7t^2}$ (a) Calculate $a_1 * a_2$.
(b) Calculate a_1/a_2.
(c) Calculate $\frac{d}{dt}(a_3)$.

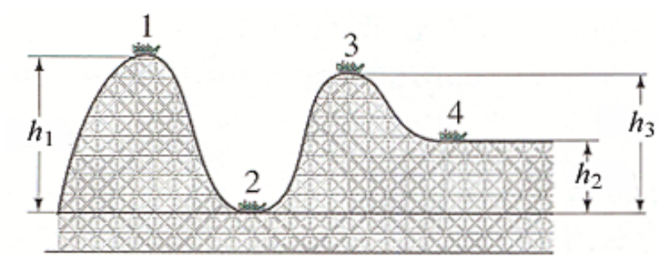
(d) Write e^x as a power series (Taylor expansion). Write the first 5 terms and a general expression.
(e) Repeat for $\sin x$
(f) Repeat for $\cos x$

21. (Review) Well behaved functions are those which are smooth and continuous.

(a) Give two examples of smooth and continuous functions. For each, provide an equation and sketch the function.
(b) Give an example of a function that isn't continuous. Sketch this function.
(c) Give an example of a function that is continuous, but not smooth. Sketch this function.

22. (Review). A cart of mass 0.3kg can slide along a frictionless track, as shown in the figure below. Here, $h_1 = 5\text{m}$, $h_2 = 2\text{m}$, and $h_3 = 4m$.

(a) Calculate the kinetic energy it needs to have at point 2 in order for it to just make it over the hill (past point 3).
(b) Describe what happens if the cart's kinetic energy is less than your answer to part a.



23. Consider $\Psi = A + iB$, where A and B are real numbers. Show that $\Psi^*\Psi$ is necessarily real and positive.

24. Use Euler's formula to show the following
(a) $\cos(\theta) = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$
(b) $\sin(\theta) = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$
Keep in mind that $e^{\pm}ix = \cos x \pm i \sin x$.

25. A particle is confined to a region $x : 0 \to L$. There is no probability that the particle is outside this region. The potential in this region is such that the wavefunction is given by

$$\Psi(x,t) = \sqrt{30/L^5}(L-x)e^{-i\omega t}$$

(a) Draw the wavefunction at $t = 0$. You're welcome to choose a value for L.

In the following, keep L and t as variables. Calculate the probability of finding the particle between
(b) 0 and L
(c) 0 and $L/4$
(d) 0 and $L/2$
(e) $L/4$ and $3L/4$

26. (Review)

$$\int \sin\left(\frac{n\pi x}{L}\right) dx =$$

Determine the indefinite integral. (We'll be using various limits.) My mistake. See next problem.

27. (Review)

$$\int \sin^2\left(\frac{n\pi x}{L}\right) dx =$$

Determine the indefinite integral. (We'll be using various limits.)

28. (Review. Maybe.) Arielle plays roulette, where the ball can land in any one of 38 possible slots. Slots can either be red or black, and there are as many black as even. (If you don't know what roulette is, look it up.)

She bets $100 that the ball will land on numbers 1-6. If she's right, which has probability of 6/38, she wins $500. If she's wrong, which has a probability of 32/38, she loses her $100. The probability distribution is

won or lost	probability
$500	6/38
-$100	32/38

Arielle plays this way 100 times. What is her average win (or loss) per game? (Final answer: is -$5.26, for a total of -$526 for 100 games. Problem from Bowling Green State University, Math)

29. (Review. Maybe.) The probability that Charles sees X number of red cars on the way to class each morning is

number of red cars	probability
0	0.41
1	0.37
2	0.16
3	0.05
4	0.01

On average, how many red cars does Charles see on the way to class? (Final answer: 0.88. Problem from wyzant.com)

30. (Review. Maybe.) The speed of cars passing by campus is measured over a 10 minutes. The distribution of these speeds are:

speed ($\pm$2.5mph)	number of cars at tobserved in 5 minutes
10	1
20	0
25	1
30	10
35	20
40	15
45	1
50	1
60	1
70	1

Calculate the (a) average speed $\overline{v}$ and the (b) average speed squared, $\overline{v^2}$.

31. (Review. Maybe.) The position of a particle is measured 10 times. Fill in the following:

data point	x	x^2
1	11	121
2	-2	4
3	6	
4	1	
5	3	
6	-10	
7	-4	
8	1	
9	-1	
10	-5	
average	$\overline{x} =$	$\overline{x^2} =$

32. (Review)

(a)

$$\int_0^L x \sin^2\left(\frac{n\pi x}{L}\right) dx =$$

(b)

$$\int_0^L x^2 \sin^2\left(\frac{n\pi x}{L}\right) dx =$$

(c)

$$\int_0^L \sin\left(\frac{n\pi x}{L}\right) \cos\left(\frac{n\pi x}{L}\right) dx =$$

33. An electron has a kinetic energy of $E = 5\text{eV}$ when it is in a region with $U = 0$. It encounters a potential step U_0.

Calculate the energy difference $(E - U_0)$ that results in reflection probability of 0.25.

(The answer is $0.11\text{E} = 0.55\text{eV}$.)

34. (Review. Maybe.) (a) Sketch a graph of hyperbolic sine, $\sinh(x)$. Write $\sinh(x)$ in terms exponentials (e^x).

(b) Repeat for hyperbolic cosine, $\cosh(x)$.

35. Consider the ground state of hydrogen,

$$R(r) = Ae^{-r/a_0}$$

The radial probability density is $4\pi r^2 R^2(r)$. The probability of finding the electron is

$$P = \int_{r_0}^{r_f} 4\pi r^2 R^2(r) dr$$

(a) Show that the normalization constant $A = 1/\sqrt{\pi a_0^3}$

(b) Show that the expectation value for the radial position $\bar{r} = 3a_0/2$

Keep in mind that the expectation value for position in 1D is $\bar{x} = \int_{\text{all space}} x\Psi^*\Psi dx$ where $\Psi^*\Psi$ is the probability density. Therefore, $\bar{r} = \int_0^\infty r[4\pi r^2 R^2(r)]dr$.

36. (a) Calculate the angular momentum of the Earth as it orbits the Sun. Given the that distance of the Earth from the Sun is much larger the radius of the Earth, model the Earth as a point particle. (The answer: $2.67 \times 10^{40}\text{kg} \cdot \text{m}^2/\text{s}$)

(b) Calculate the angular momentum of the Earth due to its rotation around an axis through its north and south poles. Model the earth as a uniform, solid sphere. (The answer: $7.1 \times 10^{33}\text{kg} \cdot \text{m}^2/\text{s}$)