HW 11/13

6.32) $U_0 = 4 \cdot 10^8 \text{ J/kg} \cdot 65 \text{ kg} = 2.6 \cdot 10^{10} \text{ J}$

$E = \frac{1}{2} m v^2 = \frac{1}{2}(65 \text{ kg})(4 \text{ m/s})^2 = 520 \text{ J}$ $\Rightarrow$ wide barrier

$$T = 16 \frac{E}{U_0}\left(1 - \frac{E}{U_0}\right) \exp\left(-2 \frac{\sqrt{2m(U_0 - E)}}{\hbar} L\right) =$$

$$= 16\left(\frac{520 \text{ J}}{2.6 \cdot 10^{10} \text{ J}}\right)\left(1 - \frac{520}{2.6 \cdot 10^{10}}\right) \exp\left(-2 \frac{\sqrt{2 \cdot 65 \cdot (2.6 \cdot 10^{10} - 520)}}{1.055 \cdot 10^{-34}} \cdot 6 \cdot 10^{11}\right)$$

$$\boxed{= e^{-2 \cdot 10^{52}} \approx 0}$$

6.33) a) $T = 16 \frac{E}{U_0}\left(1 - \frac{E}{U_0}\right) e^{-2L\sqrt{2m(U_0 - E)}/\hbar} = 16 \frac{E}{U_0}\left(1 - \frac{E}{U_0}\right) e^{-\sqrt{1 - E/U_0}\, 2L\sqrt{2mU_0}/\hbar}$

$$= 16 \frac{E}{U_0}\left(1 - \frac{E}{U_0}\right) e^{-5\sqrt{1 - E/U_0}}$$

$T_{0.4} = 16(0.4)(1 - 0.4) e^{-5\sqrt{1 - 0.4}} = \boxed{0.08}$

$T_{0.6} = 16(0.6)(0.4) e^{-5\sqrt{0.4}} = \boxed{0.16}$

b) $T_{0.4} = 16(0.4)(0.6) e^{-50\sqrt{0.6}} = \boxed{5.8 \cdot 10^{-17}}$

$T_{0.6} = 16(0.6)(0.4) e^{-50\sqrt{0.4}} = \boxed{7.1 \cdot 10^{-14}}$

c) $T_{0.4} = 16(0.4)(0.6) e^{-500\sqrt{0.6}} = \boxed{2.4 \cdot 10^{-168}}$

$T_{0.6} = 16(0.6)(0.4) e^{-500\sqrt{0.4}} = \boxed{1.8 \cdot 10^{-137}}$

d) When T is large, the higher energy has about twice the tunneling probability. At small T values, it is more than 30 magnitudes more likely to tunnel at the higher energy.

6.53) a) $\frac{d}{dx}(x^2-3)e^{-x^2} = \left[(x^2-3)(-2x)+2x\right]e^{-x^2} = (-2x^2+8)xe^{-x^2}$

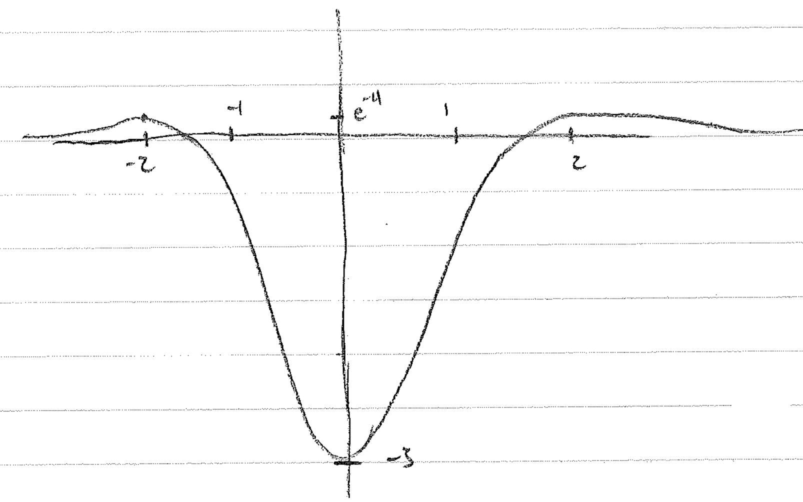
$(8-2x^2)xe^{-x^2} = 0 \quad \to x=0$

$\to x = \pm\infty$

$\to 8-2x^2=0$

$x^2=4 \quad x=\pm 2$

- at $x=\pm\infty$, $U(x)$ is 0
- at $x=0$, $U(x)$ is -3
- at $x=\pm 2$, $U(x)$ is e^{-4}



b) Only if it is "below the walls" on either side and unable to tunnel – the walls do not drop to lower than the total energy – would it be bound indefinitely.
E must be no greater than 0.

c) If it is below the walls but above the level where the walls drop farther out, it would be bound classically but quantum-mechanically tunnel: between 0 and e^{-4}

d) Yes. Even if above the tops of the walls, there is the quantum-mechanical possibility of reflection at the potential energy changes, so it could bounce back and forth for some time.