HW 11/15

7.54) $P(r) \propto r^{2n} e^{-2r/na_0}$

$$\frac{d}{dr}P(r) = \frac{d}{dr}\left(r^{2n} e^{-2r/na_0}\right) = \left(2nr^{2n-1} - r^{2n}\frac{2}{na_0}\right)e^{-2r/na_0} = 0$$

$$2nr^{2n-1} = r^{2n} \cdot \frac{2}{na_0}$$

$$n^2 a_0 = \frac{r^{2n}}{r^{2n-1}} = r^{2n-2n+1} = r \Rightarrow \boxed{r = n^2 a_0}$$

ep35) a) $R(r) = Ae^{-r/a_0}$ $\qquad P = \int_{r_0}^{r_F} 4\pi r^2 R^2(r)\, dr$

$$P = 1 = \int_0^\infty 4\pi r^2 A^2 e^{-2r/a_0} dr = 4\pi A \int_0^\infty r^2 e^{-2r/a_0} dr \quad w = -2r/a_0,\ dw = \frac{-2dr}{a_0} \quad \to r = \frac{-wa_0}{2} \to r^2 = \frac{w^2 a_0^2}{4} \quad \to 4\pi A^2 \int \frac{w^2 a_0^3}{4} \frac{e^w}{-2} dw$$

$u = w^2 \quad v = e^w$

$du = 2w\,dw \quad dv = e^w dw$

$$-\frac{A^2 a_0^3 \pi}{2}\left(w^2 e^w - \int 2we^w dw\right)$$

$u = w \quad v = e^w$

$du = dw \quad dv = e^w dw$

$$\to -\frac{A^2 a_0^3 \pi}{2}\left(w^2 e^w - 2we^w + 2\int e^w dw\right) = -\frac{A^2 a_0^3 \pi}{2}\left(w^2 e^w - 2we^w + 2e^w\right)\Bigg|_{r=0}^{r=\infty}$$

$$= \frac{-A^2 a_0^3 \pi}{2}\left[\left(\frac{r^2}{a_0^2} + 2\frac{r}{a_0} + 2\right)e^{-r/a_0}\right]_0^\infty = \frac{-A^2 a_0^3 \pi}{2}\left(0 - (0+0+2)\right) = 1$$

$$-\frac{A^2 a_0^3 \pi}{2}(-2) = A^2 a_0^3 \pi = 1$$

$$A^2 = \frac{1}{a_0^3 \pi}$$

$$A = \frac{1}{\sqrt{a_0^3 \pi}} \quad \checkmark$$

b) $$\bar{r} = \int_0^\infty r\left(4\pi r^2 R^2(r)\right)dr = \int_0^\infty 4\pi r^3\left(A^2 e^{-2r/a_0}\right)dr = 4\pi A^2 \int_0^\infty r^3 e^{-2r/a_0}\,dr$$

$r^3 = \frac{-w^3 a_0^3}{8}$ ←

$w = \frac{-2r}{a_0}$

$dw = \frac{-2dr}{a_0}$

$u = w^3 \quad v = e^w$

$du = 3w^2 dw \quad dv = e^w dw$

$$= 4\pi A^2 \int \frac{-w^3 a_0^3}{8} \cdot \frac{a_0}{-2} e^w\,dw = \frac{A^2 a_0^4 \pi}{4}\int w^3 e^w\,dw$$

$$= \frac{A^2 a_0^4 \pi}{4}\left(w^3 e^w - 3\int w^2 e^w\,dw\right)$$

* From part (a)

$$\int w^2 e^w\,dw = \left[w^2 e^w - 2we^w + 2e^w\right] = \left[\left(\frac{r^2}{a_0} + 2\frac{r}{a_0} + 2\right)e^w\right]$$

$$\bar{r} = \frac{A^2 a_0^4 \pi}{4}\left(w^3 e^w - 3w^2 e^w + 6we^w - 6e^w\right)$$

$$= \frac{A^2 a_0^4 \pi}{4}\left[\left(\frac{-8r^3}{a_0^3} - \frac{12r^2}{a_0^2} - \frac{12r}{a_0} - 6\right)e^{-2r/a_0}\right]_0^\infty$$

$$= \frac{A^2 a_0^4 \pi}{4}\left(0 - (0 - 0 - 0 - 6)\right) = \frac{3}{2}A^2 a_0^4 \pi$$

$$\Rightarrow A = \frac{1}{\sqrt{\pi a_0^3}}$$

$$\bar{r} = \frac{3}{2}\left(\frac{1}{\pi a_0^3}\right) a_0^4 \pi = \frac{3}{2} a_0 \Rightarrow \boxed{\bar{r} = \frac{3a_0}{2}} \checkmark$$

ep36)

a)

$L = I\omega$ $\omega = \frac{v}{r}$ $I = mr^2$ $\Rightarrow L = mvr$ $v = \frac{c}{t} = \frac{2\pi r}{t}$

$m_E = 5.97 \cdot 10^{24}$ kg

$r = 1.496 \cdot 10^{11}$ m

$t = 3.154 \cdot 10^{7}$ s/yr

$$L = \frac{2\pi r^2 m}{t}$$

$$L = \frac{2\pi(1.496 \cdot 10^{11} m)^2 (5.97 \cdot 10^{24} kg)}{3.154 \cdot 10^{7} s} = \boxed{2.67 \cdot 10^{40} \ kgm^2/s}$$ ✓

b)

$I_{sphere} = \frac{2}{5} mr^2$

$\omega = \frac{v}{r} = \frac{2\pi r}{tr} = \frac{2\pi}{t}$

$\Rightarrow L = I\omega = \frac{4\pi}{5} \frac{mr^2}{t}$

$t = 24$ hrs $= 86400$ s

$r = 6.371 \cdot 10^{6}$ m

$$L = \frac{4\pi}{5} \frac{(5.97 \cdot 10^{24} kg)(6.371 \cdot 10^{6} m)^2}{86,400 s} = \boxed{7.1 \cdot 10^{33} \ kg\, m^2/s}$$ ✓