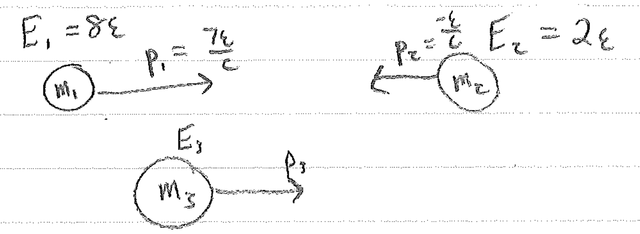
HW 9/15

1.31)



a) Particle 3 contains the total energy & momentum after the collision, so it must be the total energy & momentum

$E_3 = E_1 + E_2 = 10\varepsilon$

$p_3 = p_1 + p_2 = \frac{6\varepsilon}{c}$

b) $\frac{pc}{E} = \frac{\gamma muc}{\gamma mc^2} = \frac{u}{c}$ $\qquad \frac{u_1}{c} = \frac{7}{8}$

$\frac{u_1}{c} = \frac{p_1 c}{E_1} = \frac{\frac{7\varepsilon}{c} \cdot c}{8\varepsilon} = \frac{7}{8}$ $\qquad \frac{u_2}{c} = \frac{-1}{2}$

$\frac{u_3}{c} = \frac{3}{5}$

c) $E = \gamma mc^2$ $\qquad$ · We can compute γ from the velocities in

$m = \frac{E}{\gamma c^2}$ $\qquad$ part b. $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ $\qquad \gamma_1 = 2.066$

$\gamma_2 = 1.155$

$\gamma_1 = \frac{1}{\sqrt{1-\left(\frac{7}{8}\right)^2}}$ $\qquad \gamma_3 = 1.25$

$m_1 = \frac{E_1}{\gamma_1 c^2} = \frac{8\varepsilon}{2.066c^2} = 3.87 \frac{\varepsilon}{c^2} = m_1$

$m_2 = 1.73\ \varepsilon/c^2$

$m_3 = 8.00\ \varepsilon/c^2$

d) $m_1 + m_2 \neq m_3$

Mass is not conserved

e) Since S' is moving to the right in S frame, S is moving left in S' frame. $\Rightarrow$ $v = -0.6c$

$$\beta_u' = \frac{\beta_u - \beta_v}{1 + \beta_u \beta_v} \qquad \beta_1' = \frac{\frac{7}{8}c + \frac{-3}{5}c}{1 + \frac{7}{8}\cdot\frac{-3}{5}} = 0.579c \rightarrow \gamma_1' = 1.226$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \beta_2' = \frac{\frac{-1}{2}c + \frac{-3}{5}c}{1 + \frac{-1}{2}\cdot\frac{-3}{5}} = -0.846c \longrightarrow \gamma_2' = 1.876$$

$\hookrightarrow$ different γ, β for 1, 2, and 3

$$\beta_3' = \frac{\frac{3}{5}c + \frac{-3}{5}c}{1 + \frac{3}{5}\cdot\frac{-3}{5}} = 0 \longrightarrow \gamma_3' = 1.000$$

$\uparrow$ velocities

f)
$$E_1' = \gamma_1' m_1 c^2 = 4.75\,\varepsilon$$
$$E_2' = \gamma_2' m_2 c^2 = 3.25\,\varepsilon \quad \rightarrow \quad E_1' + E_2' = E_3' \checkmark$$
$$E_3' = \gamma_3' m_3 c^2 = 8\,\varepsilon$$

$$p_1' = \gamma_1' m_1 u_1' = 2.75\,\varepsilon/c^2$$
$$p_2' = \gamma_2' m_2 u_2' = -2.75\,\varepsilon/c^2 \quad \rightarrow \quad p_1' + p_2' = p_3' \checkmark$$
$$p_3' = \gamma_3' m_3 u_3' = 0$$

Energy and Momentum are conserved!

1.36) $m = 3\ \text{MeV}/c^2$

$E = 5\ \text{MeV}$

$$E^2 - p^2c^2 = m^2c^4$$
$$p = \sqrt{E^2 - m^2c^4}$$

$$pc = \sqrt{(5\,\text{MeV})^2 - \left(\frac{3\,\text{MeV}}{c^2}\right)^2 \cdot c^4} = \sqrt{25\,\text{MeV} - 9\,\text{MeV}} = \sqrt{16}\ \text{MeV}$$

$$= \boxed{4\ \text{MeV}}$$

$\uparrow$

c's cancel

HW 9/15

1.44) a) $E_1 = E_2 + E_3$

$20\varepsilon = 8\varepsilon + E_3$

$\boxed{E_3 = 12\varepsilon}$

b) $p_1 = p_2 + p_3$

$12\varepsilon/c = 4\varepsilon/c + p_3$

$\boxed{p_3 = 8\varepsilon/c}$

c) $\frac{pc}{E} = \frac{\gamma muc}{\gamma mc^2} = \frac{u}{c} \Rightarrow u_1 = \frac{p_1 c^2}{E_1}$

$u_1 = \frac{12\varepsilon/c \cdot c^2}{20\varepsilon} = \boxed{\frac{3}{5}c}$

d) $E_1 = \gamma_1 m_1 c^2 \Rightarrow m_1 = \frac{E_1}{\gamma_1 c^2}$

$\gamma_1 = \frac{1}{\sqrt{1-\left(\frac{3}{5}\right)^2}} = 1.25$

$m_1 = \frac{20\varepsilon}{1.25c^2} = \boxed{16\,\varepsilon/c^2}$

e) S: Frame given initially

S': rest frame of particle 1

$v = -u_1 = \frac{-3}{5}c$

$\beta = \frac{-3}{5}$

$\gamma = 1.25$

$$\begin{bmatrix} E_1' \\ p_1'c \end{bmatrix} = \begin{bmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{bmatrix} \begin{bmatrix} E_2 \\ p_2c \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 5 & -3 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 8\varepsilon \\ 4\varepsilon \end{bmatrix} = \begin{bmatrix} 7\varepsilon \\ -\varepsilon \end{bmatrix}$$