HW 10/9

4.20) $KE = \frac{p^2}{2m}$ $\quad \lambda = \frac{h}{p} \quad \Rightarrow \quad \lambda = \frac{h}{\sqrt{2m \cdot KE}}$

$p = \sqrt{2m \cdot KE}$

1 MeV: $\lambda_1 = \frac{6.626 \cdot 10^{-34}\ J \cdot s}{\sqrt{2(1.67 \cdot 10^{-27}\ kg)(1.6 \cdot 10^{-13}\ J)}} = \boxed{2.87 \cdot 10^{-14}\ m}$

20 eV: $\lambda_2 = \frac{6.626 \cdot 10^{-34}\ J \cdot s}{\sqrt{2(1.67 \cdot 10^{-27}\ kg)(20 \cdot 1.6 \cdot 10^{-19}\ J)}} = \boxed{6.41 \cdot 10^{-12}\ m}$

4.21) a) 300K: $\lambda = \frac{h}{\sqrt{3mk_BT}} = \frac{6.626 \cdot 10^{-34}\ J \cdot s}{\sqrt{3(1.67 \cdot 10^{-27}\ kg)(1.38 \cdot 10^{-23}\ J/k)(300k)}} = \boxed{1.46 \cdot 10^{-10}\ m}$

0.01c: $\lambda = \frac{h}{p} = \frac{6.626 \cdot 10^{-34}\ J \cdot s}{(1.67 \cdot 10^{-27})(3 \cdot 10^{6}\ m/s)} = \boxed{1.32 \cdot 10^{-13}\ m}$

b) electron at 300K: $\lambda = \frac{h}{\sqrt{3mk_BT}} = \frac{6.626 \cdot 10^{-34}\ J \cdot s}{\sqrt{3(9.11 \cdot 10^{-31}\ kg)(1.38 \cdot 10^{-23}\ J/k)(300k)}} = \boxed{6.23 \cdot 10^{-9}\ m}$

0.01c: $\lambda = \frac{h}{p} = \frac{6.626 \cdot 10^{-34}\ J \cdot s}{(9.11 \cdot 10^{-31}\ kg)(3 \cdot 10^{6}\ m/s)} = \boxed{2.43 \cdot 10^{-10}\ m}$

c) For the electron, dimensions smaller than 6nm down to 0.2nm. For the neutron, smaller than 0.1nm to 0.0001 nm. The range is smaller for the electron because with a small mass, it is already moving fast even at room temperature. The electron's smaller mass also accounts for its larger wavelength, and thus the greater likelihood that it will reveal a wave nature.

4.27) a) $\lambda = \frac{h}{\sqrt{2mqV}}$ → from example 4.3

$$\lambda = \frac{6.626 \cdot 10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(9.11 \cdot 10^{-31}\ \text{kg})(1.6 \cdot 10^{-19}\ \text{C})(20\text{V})}} = 2.75 \cdot 10^{-10}\ \text{m}$$

$$\frac{1}{2}\lambda = d\sin\theta \approx d\frac{y}{L} \implies y = \frac{\lambda L}{2d}$$

$$y = \frac{(2.75 \cdot 10^{-10}\ \text{m})(10\ \text{m})}{2(10^{-5}\ \text{m})} = \boxed{0.14\ \text{mm}}$$

b) $\psi_1 \propto \sqrt{100} = 10$ → $\psi_{total} \propto \psi_1 + \psi_2 = 40$

$\psi_2 \propto \sqrt{900} = 30$

$$|\psi_{total}|^2 = \boxed{1600}$$

c) $\psi_{total} \propto \psi_2 - \psi_1 = 20$

$$|\psi_{total}|^2 = \boxed{400}$$

4.71) a) $(\gamma_u - 1)mc^2 = mc^2 \rightarrow \gamma_u = 2 \rightarrow u = \frac{\sqrt{3}}{2}c$

$$p = \gamma_u m u = 2(1.67 \cdot 10^{-27}\ \text{kg})\left(\frac{\sqrt{3}}{2}c\right) = 8.68 \cdot 10^{-19}\ \text{kg m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.626 \cdot 10^{-34}\ \text{J}\cdot\text{s}}{8.68 \cdot 10^{-19}\ \text{kg m/s}} = \boxed{7.64 \cdot 10^{-16}\ \text{m}}$$

b) This is smaller than the proton's approximate radius, so a proton this fast will behave as a particle.

ep18) a) $\phi_n = \frac{n}{\Delta t\, \Delta A}$

$$I = hf\phi = \frac{hc}{\lambda}\phi$$

$$I_n = \frac{hc}{\lambda}\phi_n = \frac{hc}{\lambda}\cdot\frac{n}{\Delta t\, \Delta A} = \frac{hc}{\lambda \Delta A}\cdot\frac{n}{\Delta t}$$

$$\hookrightarrow \frac{hc}{\lambda \Delta A} = \frac{(6.626\cdot 10^{-34}\,Js)(3\cdot 10^{8}\,m/s)}{(650\cdot 10^{-9}\,m)(5\cdot 10^{-5}\,m)^2\pi} = 3.8938\cdot 10^{-11}\ W/m^2$$

So then for $\frac{n}{\Delta t} = 0, 10, 30, 40$:

$$I_0 = 0$$
$$I_{10} = 3.8938\cdot 10^{-10}\ W/m^2$$
$$I_{30} = 30\cdot 3.89\cdot 10^{-11} = 1.168\cdot 10^{-9}\ W/m^2$$
$$I_{40} = 40\cdot 3.89\cdot 10^{-11} = 1.557\cdot 10^{-9}\ W/m^2$$

b) $I = \frac{1}{2}c\varepsilon_0 E_0^2 \rightarrow E_n = \sqrt{\frac{2I_n}{c\varepsilon_0}}$

$$E_0 = 0$$
$$E_{10} = \sqrt{\frac{2I_{10}}{c\varepsilon_0}} = \sqrt{\frac{2(3.89\cdot 10^{-10}\,W/m^2)}{(3\cdot 10^{8}\,m/s)(8.854\cdot 10^{-12})}} = 5.414\cdot 10^{-4}\ N/m$$

$$E_{30} = 9.378\cdot 10^{-4}\ N/m$$
$$E_{40} = 0.00108\ N/m$$

c) E_s = field strength of a single slit
At the interference maximum with two slits, the field strength will be $2E_s$. And since the two slit max is E_{40}, $E_s = \frac{1}{2}E_{40} \rightarrow \boxed{E_s = 5.414\cdot 10^{-4}\ N/m}$

d) Since we see that $E_s = E_{10}$, 10 photons/s arrive at each slit.