HW 10/13

4.52) a) $E = KE + PE$

$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \rightarrow \boxed{E = \frac{p^2}{2m} + \frac{1}{2}kx^2}$

b) $E = \frac{(\Delta p)^2}{2m} + \frac{1}{2}k(\Delta x)^2 \qquad \Delta p \geq \frac{\hbar}{2\Delta x}$

$\rightarrow \boxed{E = \frac{(\hbar/2\Delta x)^2}{2m} + \frac{1}{2}k(\Delta x)^2}$

c) $\frac{dE}{d\Delta x} = \frac{-\hbar^2}{4m(\Delta x)^3} + k\Delta x = 0$

$k\Delta x = \frac{\hbar^2}{4m(\Delta x)^3}$

$(\Delta x)^4 = \frac{\hbar^2}{4km} \rightarrow \Delta x = \left(\frac{\hbar^2}{4km}\right)^{1/4}$

$E = \frac{\hbar^2}{8m(\Delta x)^2} + \frac{1}{2}k(\Delta x)^2$

$\hookrightarrow E = \frac{\hbar^2}{8m\left(\frac{\hbar^2}{4km}\right)^{1/2}} + \frac{1}{2}k\left(\frac{\hbar^2}{4km}\right)^{1/2}$

$E = \frac{\hbar^2}{4m^{1/2}\hbar/\sqrt{k}} + \frac{\sqrt{k}\hbar}{2\cdot 2m^{1/2}} = \frac{\sqrt{k}\hbar}{4\sqrt{m}} + \frac{\sqrt{k}\hbar}{4\sqrt{m}} = \frac{2\sqrt{k}\hbar}{4\sqrt{m}} = \boxed{\frac{\hbar}{2}\sqrt{\frac{k}{m}}}$

ep19) $\vec{A} = 5\hat{x} - 6\hat{y}$

$\vec{B} = -3\hat{x} + 2\hat{y}$

a) $\vec{A} + \vec{B} = 5\hat{x} - 6\hat{y} - 3\hat{x} + 2\hat{y} = \boxed{2\hat{x} - 4\hat{y} = \vec{C}}$

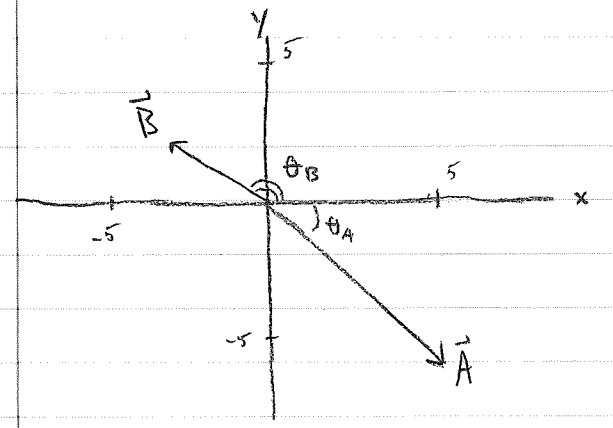
b) $|\vec{A}| = \sqrt{5^2 + (-6)^2} = \sqrt{25+36} = \boxed{\sqrt{61}}$

direction: $\tan\theta_A = \frac{y}{x} \rightarrow \theta_A = \tan^{-1}\left(\frac{-6}{5}\right) = -50.19^\circ \Longrightarrow \boxed{\theta_A = 50.19^\circ}$

c) $|\vec{B}| = \sqrt{(-3)^2 + 2^2} = \sqrt{9+4} = \boxed{\sqrt{13}}$

$\theta_B = \tan^{-1}\left(\frac{y}{x}\right) \rightarrow \theta_B = -33.69^\circ \xrightarrow{+180^\circ} \boxed{\theta_B = -146.3^\circ}$

d)



ep20) $a_1 = 6e^{3t}$ $a_2 = 3e^{-4t}$ $a_3 = 5e^{-7t^2}$

a) $a_1 \times a_2 = 6e^{3t} \cdot 3e^{-4t} = 18e^{3t-4t} = \boxed{18e^{-t}}$

b) $\frac{a_1}{a_2} = \frac{6e^{3t}}{3e^{-4t}} = 2e^{3t-(-4t)} = \boxed{2e^{7t}}$

c) $\frac{d}{dt}(a_3) = \frac{d}{dt}\left(5e^{-7t^2}\right) = 5\left(e^{-7t^2} \cdot (-14t)\right) = \boxed{-70te^{-7t^2}}$

d) $e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$

$e^x = \sum_{n=1}^{\infty} \frac{x^n}{n!}$

e) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$

f) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$