HW 10/18

4.37) $\Psi(x,t) = A\exp\left[i\left(1.58\cdot10^{12}x - 7.91\cdot10^{16}t\right)\right]$

$\Psi = Ae^{i(kx-\omega t)}$ $\quad k = 1.58\cdot10^{12}$

$\omega = 7.91\cdot10^{16}$

$p = \hbar k = (1.055\cdot10^{-34}\ Js)(1.58\cdot10^{12}\ \frac{1}{m}) = \boxed{1.67\cdot10^{-22}\ kg\ m/s}$

$E = \hbar\omega = (1.055\cdot10^{-34}\ Js)(7.91\cdot10^{16}\ 1/s) = \boxed{8.35\cdot10^{-18}\ J}$

$E = KE = \frac{p^2}{2m}$

$\hookrightarrow m = \frac{p^2}{2E} = \frac{(1.67\cdot10^{-22}\ kg\ m/s)^2}{2(8.35\cdot10^{-18}\ J)} = \boxed{1.66\cdot10^{-27}\ kg}$

ep23) $\Psi = A + Bi \qquad \Psi^* = A - Bi$

$\Psi^*\Psi = (A-Bi)(A+Bi) = A^2 - B^2i^2 = A^2 + B^2$

→ Since A & B are real, A^2 and B^2 are real and positive. Thus $\Psi^*\Psi = A^2 + B^2$ is real and positive

ep24) a) $\cos(\theta) = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$ $\qquad e^{i\theta} = \cos\theta + i\sin\theta$

$\cos\theta = \cos(-\theta)$

$\sin(-\theta) = -\sin\theta$

$\frac{1}{2}(e^{i\theta} + e^{-i\theta}) = \frac{1}{2}(\cos\theta + i\sin\theta + \cos(-\theta) + i\sin(-\theta))$

$= \frac{1}{2}(\cos\theta + \cos\theta + i\sin\theta - i\sin\theta)$

$= \frac{1}{2}(2\cos\theta)$

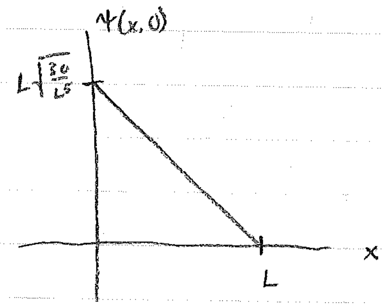
$\frac{1}{2}(e^{i\theta} + e^{-i\theta}) = \cos\theta$ ✓

ep24) b) $\frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right) = \frac{1}{2i}\left(\cos\theta + i\sin\theta - \cos\theta + i\sin\theta\right)$

$$= \frac{1}{2i}\left(2i\sin\theta\right)$$

$$\frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right) = \sin\theta \quad \checkmark$$

ep25) $\Psi(x,t) = \sqrt{\frac{30}{L^5}}\,(L-x)\,e^{-i\omega t}$



a) $t=0$ $\qquad \Psi(x,0) = \sqrt{\frac{30}{L^5}}\,(L-x)$

b) $\text{Prob}(0<x<L) = \int_0^L \Psi^*\Psi\,dx = \int_0^L \sqrt{\frac{30}{L^5}}(L-x)e^{-i\omega t}\sqrt{\frac{30}{L^5}}(L-x)e^{i\omega t}\,dx$

$$= \frac{30}{L^5}\int_0^L (L-x)^2\,dx = \frac{30}{L^5}\int_0^L L^2 - 2Lx + x^2\,dx = \frac{30}{L^5}\left[L^2x - Lx^2 + \frac{x^3}{3}\right]_0^L$$

$$= \frac{30}{L^5}\left(L^3 - L^3 + \frac{L^3}{3} - 0\right) = \frac{30}{L^5}\cdot\frac{L^3}{3} = \boxed{\frac{10}{L^2}}$$

c) $\text{Prob}\left(0 < x < \frac{L}{4}\right) = \int_0^{L/4} \Psi^*\Psi\,dx = \frac{30}{L^5}\left[L^2x - Lx^2 + \frac{x^3}{3}\right]_0^{L/4} = \frac{30}{L^5}\left[\frac{L^3}{4} - \frac{L^3}{16} + \frac{L^3}{192}\right]$

$$= \frac{30}{L^5}\left[\frac{48L^3 - 12L^3 + L^3}{192}\right] = \frac{30}{L^5}\left(\frac{37}{192}L^3\right) = \boxed{\frac{185}{32L^2}}$$

d) $\text{Prob}\left(0<x<\frac{L}{2}\right) = \int_0^{L/2} \Psi^*\Psi\,dx = \frac{30}{L^5}\left[L^2x - Lx^2 + \frac{x^3}{3}\right]_0^{L/2} = \frac{30}{L^5}\left[\frac{L^3}{2} - \frac{L^3}{4} + \frac{L^3}{24}\right]$

$$= \frac{30}{L^5}\left(\frac{12L^3 - 6L^3 + L^3}{24}\right) = \frac{30}{L^5}\left(\frac{7L^3}{24}\right) = \boxed{\frac{35}{4L^2}}$$

e) $\text{Prob}\left(\frac{L}{4} < x < \frac{3L}{4}\right) = \int_{L/4}^{3L/4} \Psi^*\Psi\,dx = \frac{30}{L^5}\left[L^2x - Lx^2 + \frac{x^3}{3}\right]_{\frac{L}{4}}^{3L/4} = \frac{30}{L^5}\left[\frac{3L^3}{4} - \frac{9L^3}{16} + \frac{27L^3}{192} - \frac{37L^3}{192}\right]$

$$= \frac{30}{L^5}\left(\frac{144L^3 - 108L^3 + 27L^3 - 37L^3}{192}\right) = \frac{30\cdot 26}{192L^2} = \boxed{\frac{65}{16L^2}}$$