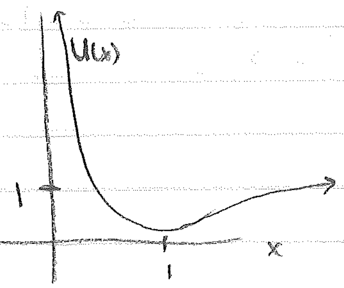
HW 10/23

5.21) a) $\lim_{x\to 0} U(x)$ goes to $+\infty$ since $\frac{+1}{x^2}$ diverges faster

$\lim_{x\to\infty} U(x)$ goes to 1



b) KE is zero when total energy $E = U$.

$E = 0.5 = \frac{1}{x^2} - \frac{2}{x} + 1 \longrightarrow 0.5x^2 - 2x + 1 = 0$

$$x = \frac{2 \pm \sqrt{2^2 - 4(0.5)}}{2(0.5)} = \boxed{2 \pm \sqrt{2}}$$

Yes the particle is bound b/c there are turning points on either side.

c) $E = U$

$2 = \frac{1}{x^2} - \frac{2}{x} + 1$

$1 + \frac{2}{x} - \frac{1}{x^2} = 0$

$x^2 + 2x - 1 = 0 \longrightarrow x = \frac{-2 \pm \sqrt{2^2 - 4(-1)}}{2} = \frac{-2 \pm \sqrt{8}}{2} \rightarrow \boxed{x = \sqrt{2} - 1}$

The only positive root is $x = \sqrt{2} - 1$, so it is not bounded

ep26) $\int \sin\left(\frac{n\pi x}{L}\right) dx$ $\quad u = \frac{n\pi x}{L}$, $du = \frac{n\pi}{L} dx$ $\quad = \frac{L}{n\pi} \int \sin u \, du = \boxed{\frac{-L}{n\pi} \cos\left(\frac{n\pi x}{L}\right) + c}$

5.23) $\Psi(x,t) = \psi(x)\,\phi(t) = \sqrt{\frac{2}{L}}\,\sin\frac{3\pi x}{L}\;e^{-i(E_3/\hbar)t}$

$$= \sqrt{\frac{2}{10^{-8}\,m}}\,\sin\left(\frac{3\pi x}{10^{-8}\,m}\right)\exp\left[-i\left(\frac{3^2\pi^2\hbar^2}{2(9.11\cdot 10^{-31}\,kg)(10^{-8}\,m)^2}\right)\cdot\frac{1}{\hbar}t\right]$$

$$= 1.41\cdot 10^{4}\,m^{-1/2}\,\sin\left(9.42\cdot 10^{8}\,m^{-1}\,x\right)e^{-i(5.14\cdot 10^{13}\,s^{-1})t}$$

5.33) $\frac{d^2\psi(x)}{dx^2} = \frac{d}{dx^2}\left(A\sin(kx) + B\cos(kx)\right)$

$$= \frac{d}{dx}\left(kA\cos(kx) - kB\sin(kx)\right)$$

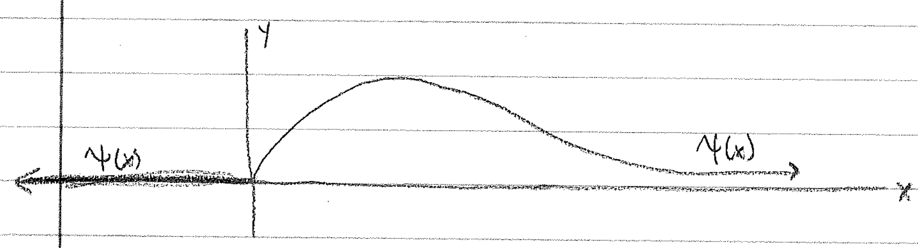
$$= -k^2 A\sin(kx) - k^2 B\cos(kx)$$

$$\frac{d^2\psi(x)}{dx^2} = -k^2\,\psi(x)$$

5.78) $\int_{-\infty}^{\infty}\psi^2\,dx = \int_0^{\infty}\left(2\sqrt{a^3}\,x e^{-ax}\right)^2 dx = 4a^3\int_0^{\infty} x^2 e^{-2ax}\,dx = 4a^3\,\frac{2!}{(2a)^3}$ → (using table from front of book)

$$= 4a^3\,\frac{2}{8a^3} = 1 \quad \checkmark$$

5.79)



It is smooth except at $x = 0$