HW 10/25

5.25) Energy levels get further apart as n increases, so the lowest energy transition will be from $n=2 \rightarrow n=1$.

$E_{photon} \Rightarrow hf = h^{c}/_{\lambda}$

$E_2 - E_1 = \frac{\pi^2 \hbar^2}{2mL^2}(2^2 - 1^2)$

$\frac{hc}{\lambda} = \frac{3\pi^2\hbar^2}{2mL^2} \longrightarrow L^2 = \frac{3\pi^2\hbar^2\lambda}{2mhc} \longrightarrow L = \pi\hbar\sqrt{\frac{3\lambda}{2mhc}}$

$L = \pi(1.055 \cdot 10^{-34}\ Js)\sqrt{\frac{450 \cdot 10^{-9}\ m \cdot 3}{2(9.11 \cdot 10^{-31}\ kg)(6.63 \cdot 10^{-34}\ Js)(3 \cdot 10^{8}\ m/s)}} = 6.3989 \cdot 10^{-10}\ m$

$\boxed{L = 0.64\ nm}$

5.28) $\psi_2(x) = \sqrt{\frac{2}{L}} \sin\frac{2\pi x}{L}$

$Prob = \int |\psi_2(x)|^2 dx = \frac{2}{L}\int_{\frac{L}{3}}^{\frac{2L}{3}} \sin^2\left(\frac{2\pi x}{L}\right) dx = \frac{2}{L}\left[\frac{x}{2} - \frac{L\sin\left(\frac{4\pi x}{L}\right)}{8\pi}\right]_{L/3}^{2L/3}$

$= \frac{2}{L}\left(\frac{L}{6} - L\frac{\sin\frac{8\pi}{3} - \sin\frac{4\pi}{3}}{8\pi}\right) = \boxed{0.196}$

Classically this should be 1/3. It is lower because the region is centered on a node.

5.31) (5-19) $\psi(x) = Ae^{\pm\alpha x}$ $\quad \alpha \equiv \sqrt{\frac{2m(U_0 - E)}{\hbar^2}}$

(5-18) $\frac{d^2}{dx^2}\psi(x) = \frac{2m(U(x) - E)}{\hbar^2}\psi(x)$

$\frac{d^2}{dx^2}\left(Ae^{\pm\alpha x}\right) = A\alpha^2 e^{\pm\alpha x} = \frac{2m(U_0 - E)}{\hbar^2} Ae^{\pm\alpha x} = \frac{2m(U_0 - E)}{\hbar^2}\psi(x)$ ✓

5.93) a) $\psi(x) = \dfrac{\sqrt{2/\pi}}{x^2 - x + 1.25}$

$$\int_{-\infty}^{\infty} \psi^2(x)\,dx = \int_{-\infty}^{\infty} \frac{2/\pi}{(x^2 - x + 1.25)^2}\,dx = \int_{-\infty}^{\infty} \frac{2/\pi}{(x^2+1)^2}\,dx$$

Substitution: $x = \tan\theta \longrightarrow x^2 + 1 = \tan^2\theta + 1 = \sec^2\theta$

$dx = \sec^2\theta\,d\theta$

$$\Rightarrow \frac{2}{\pi}\int_{-\infty}^{\infty} \frac{\sec^2\theta}{(\sec^2\theta)^2}\,d\theta = \frac{2}{\pi}\int_{-\infty}^{\infty} \cos^2\theta\,d\theta = \frac{2}{\pi}\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right)\Bigg|_{-\pi/2}^{\pi/2}$$

$$= \frac{2}{\pi}\left(\frac{\pi}{4} + 0 - \left(-\frac{\pi}{4}\right) + 0\right) = \frac{2}{\pi}\cdot\frac{\pi}{2} = 1 \quad \checkmark$$

b) $$\frac{d}{dx}\psi(x) = \frac{-\sqrt{2/\pi}\,(2x-1)}{(x^2 - x + 1.25)^2} = 0$$

This is zero when the numerator is $= 0$

$-\sqrt{\frac{2}{\pi}}(2x-1) = 0$

$2x - 1 = 0$

$\boxed{x = \frac{1}{2}}$

c) $$\psi^2\left(\tfrac{1}{2}\right) = \left(\frac{\sqrt{2/\pi}}{\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) + 1.25}\right)^2 = \left(\frac{\sqrt{2/\pi}}{\frac{1}{4} - \frac{2}{4} + \frac{5}{4}}\right)^2 = \sqrt{\frac{2}{\pi}}^{\,2} = \frac{2}{\pi} = \boxed{0.637}$$

ep27) $$\int \sin^2\left(\frac{n\pi x}{L}\right)dx = \int \frac{1 - \cos\left(\frac{2n\pi x}{L}\right)}{2}\,dx \qquad u = \frac{2n\pi x}{L} \qquad du = \frac{2n\pi}{L}\,dx$$

$$= \frac{1}{2}\cdot\frac{L}{2n\pi}\int 1 - \cos(u)\,du = \frac{L}{4n\pi}\Big[u - \sin u\Big] = \frac{L}{4n\pi}\left[\frac{2n\pi x}{L} - \sin\left(\frac{2n\pi x}{L}\right)\right]$$

$$= \boxed{\frac{x}{2} - \frac{L}{4n\pi}\sin\left(\frac{2n\pi x}{L}\right) + C}$$