HW 11/1

ep31)

data point	x	x^2
1	11	121
2	-2	4
3	6	36
4	1	1
5	3	9
6	-10	100
7	-4	16
8	1	1
9	-1	1
10	-5	25
total	0	314

Average:

$$\bar{x} = \frac{0}{10} = 0$$

$$\overline{x^2} = \frac{314}{10} = 31.4$$

ep32) a)

$$\int_0^L x\sin^2\left(\frac{n\pi x}{L}\right)dx \qquad u = x \quad du = dx \qquad v = \frac{x}{2} - \frac{L}{4\pi n}\sin\left(\frac{2n\pi x}{L}\right) \quad dv = \sin^2\left(\frac{n\pi x}{L}\right)dx \longrightarrow \left\{ \begin{array}{l} w = \frac{n\pi x}{L} \\ dw = \frac{n\pi}{L}dx \end{array} \right.$$

$$= \frac{x^2}{2} - \frac{Lx}{4\pi n}\sin\left(\frac{2n\pi x}{L}\right) - \int_0^L \frac{x}{2} - \frac{L}{4\pi n}\sin\left(\frac{2n\pi x}{L}\right)dx \longrightarrow \begin{array}{l} z = \frac{2n\pi x}{L} \\ dx = \frac{L}{2n\pi}dx \end{array}$$

$$= \frac{x^2}{2} - \frac{Lx}{4\pi n}\sin\left(\frac{2n\pi x}{L}\right) - \left[\frac{x^2}{4} - \frac{L^2}{8n^2\pi^2}\cos\left(\frac{2n\pi x}{L}\right)\right]\Bigg|_0^L$$

$$= \frac{L^2}{2} - \frac{L^2}{4} - \frac{L^2}{4\pi n}\sin(2\pi n) - \frac{L^2}{8n^2\pi^2}\cos(2\pi n) + \frac{L^2}{8n^2\pi^2}\overbrace{\cos(0)}^{=1}$$

$$\boxed{= \frac{L^2}{4} + \frac{L^2}{8n^2\pi^2} - \frac{L^2}{4\pi n}\sin(2\pi n) - \frac{L^2}{8\pi^2 n^2}\cos(2\pi n)}$$

b) $\int_0^L x^2 \sin^2\left(\frac{n\pi x}{L}\right) dx \longrightarrow$ $u = x^2$, $du = 2x\,dx$ $\quad$ $v = \frac{x}{2} - \frac{L}{4\pi n}\sin\left(\frac{2n\pi x}{L}\right)$, $dv = \sin^2\left(\frac{n\pi x}{L}\right)dx$

$$= \frac{x^3}{2} - \frac{Lx^2}{4\pi n}\sin\left(\frac{2n\pi x}{L}\right)\Bigg|_0^L - \int_0^L x^2 - \frac{Lx}{2\pi n}\sin\left(\frac{2n\pi x}{L}\right) dx$$

$$= \frac{L^3}{2} - \frac{L^3}{4\pi n}\sin(2\pi n) - \left[\frac{x^3}{3}\right]_0^L + \frac{L}{2\pi n}\int x\sin\left(\frac{2\pi n x}{L}\right)dx \longrightarrow$$ $u = x$, $du = dx$ $\quad$ $v = \frac{-L}{2\pi n}\cos\left(\frac{2\pi n x}{L}\right)$, $dv = \sin\left(\frac{2\pi n x}{L}\right)$

$$= \frac{L^3}{2} - \frac{L^3}{3} - \frac{L^3}{4\pi n}\sin(2\pi n) + \frac{L}{2\pi n}\left[\frac{-Lx}{2\pi n}\cos\left(\frac{2\pi n x}{L}\right)\Bigg|_0^L + \int_0^L \frac{+L}{2\pi n}\cos\left(\frac{2\pi n x}{L}\right)dx\right]$$

$$= \frac{L^3}{6} - \frac{L^3}{4\pi n}\sin(2\pi n) - \frac{L^3}{4n^2\pi^2}\cos(2\pi n) + \frac{L^3}{4\pi^2 n^2} + \frac{L}{2\pi n}\left[\left(\frac{L}{2\pi n}\right)^2\sin\left(\frac{2\pi n x}{L}\right)\right]_0^L$$

$$\boxed{= \frac{L^3}{6} + \frac{L^3}{4\pi^2 n^2} - \frac{L^3}{4\pi n}\sin(2\pi n) - \frac{L^3}{4n^2\pi^2}\cos(2\pi n) + \frac{L^3}{8\pi^3 n^3}\sin(2\pi n)}$$

c) $\int_0^L \sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\pi x}{L}\right)dx \longrightarrow$ $u = \sin\left(\frac{n\pi x}{L}\right)$, $du = \frac{n\pi}{L}\cos\left(\frac{n\pi x}{L}\right)dx$

$$= \frac{L}{n\pi}\int_{u_1}^{u_2} u\,du = \frac{L}{n\pi}\left[\frac{u^2}{2}\right]_{u_1}^{u_2} = \frac{L}{2n\pi}\left[\sin^2\left(\frac{n\pi x}{L}\right)\right]_0^L = \boxed{\frac{L}{2\pi n}\sin^2(\pi n)}$$