HW 11/3

5.82)
$$\psi(x) = \begin{cases} 2\sqrt{a^3}\, x e^{-ax} & x>0 \\ 0 & x<0 \end{cases}$$

$$\int_{\text{all space}} x\psi^2(x)\,dx = \int_0^\infty x\left(2\sqrt{a^3}\,xe^{-ax}\right)^2 dx = 4a^3\int_0^\infty x^3 e^{-2ax}$$

$$= 4a^3 \frac{3!}{(2a)^4} = \boxed{\frac{1.5}{a}}$$

5.83)
$$\bar{x} = \int x\psi^2(x)\,dx = \frac{1.5}{a} \quad \text{(from 5.82)}$$

$$\overline{x^2} = \int_{\text{all space}} x^2\psi^2(x)\,dx = \int_0^\infty x^2\left(2\sqrt{a^3}\,xe^{-ax}\right)^2 = 4a^3\int x^4 e^{-2ax}$$

$$= 4a^3\frac{4!}{(2a)^5} = \frac{3}{a^2}$$

$$\Delta x = \sqrt{\overline{x^2} - (\bar{x})^2} = \sqrt{\frac{3}{a^2} - \left(\frac{1.5}{a}\right)^2} = \sqrt{\frac{3-2.25}{a^2}} = \frac{\sqrt{0.75}}{a} = \boxed{\frac{0.866}{a}}$$

5.84)
$$\bar{p} = \int_0^\infty \left(2\sqrt{a^3}\,xe^{-ax}\right)\left(-i\hbar\frac{d}{dx}\right)\left(2\sqrt{a^3}\,xe^{-ax}\right) = 4a^3(-i\hbar)\int_0^\infty xe^{-ax}\left((1-ax)e^{-ax}\right)dx$$

$$= 4a^3(-i\hbar)\left(\int_0^\infty xe^{-2ax}dx - a\int_0^\infty x^2e^{-2ax}dx\right) = 4a^3(-i\hbar)\left(\frac{1!}{(2a)^2} - a\frac{2!}{(2a)^3}\right)$$

$$= 4a^3(-i\hbar)\left(\frac{1}{4a^2} - \frac{2a}{8a^3}\right) = 0$$

The particle is bound. It is just as likely to have positive or negative momentum.

5.85) $\bar{p} = 0$ (from 5.84)

$$\overline{p^2} = \int_0^\infty \left(2\sqrt{a^3}\,x e^{-ax}\right)\left(-i\hbar\frac{\partial}{\partial x}\right)^2\left(2\sqrt{a^3}\,x e^{-ax}\right)dx = 4a^3(-\hbar^2)\int_0^\infty \left(a^2x^2 - 2ax\right)e^{-2ax}\,dx$$

$$= -4a^3\hbar^2\left(a^2\frac{2!}{(2a)^3} - 2a\frac{1!}{(2a)^2}\right) = -4a^3\hbar^2\left(\frac{2a^2}{8a^3} - \frac{2a}{4a^2}\right)$$

$$= -4a^3\hbar^2\left(\frac{1}{4a} - \frac{2}{4a}\right) = a^2\hbar^2$$

$$\Delta p = \sqrt{\overline{p^2} - (\bar{p})^2} = \sqrt{a^2\hbar^2} = \boxed{a\hbar}$$

5.86) $\Delta x \Delta p = \frac{0.866}{a}\,a\hbar = \boxed{0.866\hbar} > 0.5\hbar$

The product is $\geq \frac{\hbar}{2}$, as it must be. Since the wave function is not a Gaussian, it should be greater than the minimum product of $\frac{\hbar}{2}$.