HW 9/1 Solutions

1.3) $S' = \begin{bmatrix} t' \\ x' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 7\,m/s & 1 \end{bmatrix} \begin{bmatrix} 2s \\ 5m \end{bmatrix} = \begin{bmatrix} 1(2s) + 0(5m) \\ 7\frac{m}{s}(2s) + 1(5m) \end{bmatrix} = \begin{bmatrix} 2s \\ 19m \end{bmatrix}$

1.4) $S' = \begin{bmatrix} t' \\ x' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -7\frac{m}{s} & 1 \end{bmatrix} \begin{bmatrix} 2s \\ 5m \end{bmatrix} = \begin{bmatrix} 1(2s) + 0(5m) \\ -7\frac{m}{s}(2s) + 1(5m) \end{bmatrix} = \begin{bmatrix} 2s \\ -9m \end{bmatrix}$

1.7) $\beta = \frac{v}{c}$

$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

v	β	γ
$10^3\ \frac{m}{s}$	$\frac{1}{3} \cdot 10^{-5}$	1.000
$10^6\ \frac{m}{s}$	$\frac{1}{3} \cdot 10^{-2}$	1.000
$0.1c$	0.1	1.005
$0.5c$	0.5	1.155
$0.9c$	0.9	2.294
$0.999c$	0.999	22.366
$\frac{3}{5}c$	$\frac{3}{5}$	$\frac{5}{4}$
$\frac{\sqrt{3}}{2}c$	$\frac{\sqrt{3}}{2}$	2
$\frac{4}{5}c$	$\frac{4}{5}$	$\frac{5}{3}$

$v = \frac{3}{5}c$: $\beta = \frac{\frac{3}{5}c}{c} = \frac{3}{5}$ $\gamma = \frac{1}{\sqrt{1 - \frac{(\frac{3}{5}c)^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{9}{25}c^2/c^2}} = \frac{1}{\sqrt{16/25}} = \frac{1}{4/5}$

$\boxed{\gamma = \frac{5}{4}}$

$v = \frac{\sqrt{3}}{2}c$: $\gamma = \frac{1}{\sqrt{1 - (\frac{\sqrt{3}}{2}c)^2/c^2}} = \frac{1}{\sqrt{1 - \frac{3}{4}c^2/c^2}} = \frac{1}{\sqrt{1/4}} = \frac{1}{1/2} = \boxed{2}$

$v = \frac{4}{5}c$: $\gamma = \frac{1}{\sqrt{1 - (\frac{4}{5}c)^2/c^2}} = \frac{1}{\sqrt{1 - \frac{16}{25}c^2/c^2}} = \frac{1}{\sqrt{9/25}} = \frac{1}{3/5} = \boxed{\frac{5}{3}}$

1.9)

$$S' = \begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix} = \begin{bmatrix} \frac{5}{3} & \frac{5}{3}\left(\frac{-4}{5}\right) \\ \frac{5}{3}\left(\frac{-4}{5}\right) & \frac{5}{3} \end{bmatrix} \begin{bmatrix} 3m \\ 5m \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{3}(3m) + \frac{-4}{3}(5m) \\ \frac{-4}{3}(3m) + \frac{5}{3}(5m) \end{bmatrix} = \begin{bmatrix} \frac{-5}{3}m \\ \frac{13}{3}m \end{bmatrix}$$