Taylor Series (Prob Calc II?)

$$f(x) = f(a) + (x-a)\frac{f'(a)}{\cancel{2}1!} + (x-a)^2\frac{f''(a)}{2!} + \ldots$$

at $x=a$

Use this to derive 2nd Order RK

lets replace $f(x) \to x(t)$

and eval at $x = \underbrace{t + \tfrac{1}{2}h}_{a}$

$$\dot{x}(t) = x\left(t+\tfrac{1}{2}h\right) + \left[t - t + \tfrac{1}{2}h\right]\dot{x}\left(t+\tfrac{1}{2}h\right)\Big|_{t+\frac{1}{2}h} + \left(-\frac{h}{2}\right)^2 \frac{\ddot{x}\left(t+\frac{1}{2}h\right)}{2} + O(h)^3$$

(1) $$x(t) \cong x\left(t+\frac{h}{2}\right) - \frac{h}{2}\dot{x}\Big|_{t+\frac{h}{2}} + \left(\frac{h}{2}\right)^2 \frac{\ddot{x}}{2}\Big|_{t+\frac{h}{2}} + O(h)^3$$

Now do the Taylor Exp for $x(t+h)$

at $a = t + \frac{h}{2}$

$$x(t+h) \cong x\left(t+\frac{h}{2}\right) + \left[t+h-\left(t+\frac{h}{2}\right)\right]\dot{x}\Big|_{t+\frac{h}{2}} + \left(\frac{h}{2}\right)^2 \ddot{x}\Big|_{t+\frac{h}{2}} + O(h^3)$$

$$② \quad x(t+h) \doteq x\left(t+\frac{h}{2}\right) + \frac{h}{2}\dot{x}\Big|_{t+\frac{h}{2}} + \left(\frac{h}{2}\right)^2 \ddot{x}\Big|_{t+\frac{h}{2}} + O(h^3)$$

② − ① — cancel or   cancel

$$x(t+h) - x(t) = 2\frac{h}{2}\dot{x}\Big|_{t+\frac{h}{2}} + 0 + O(h)^3$$

We will ignore higher order

$$\boxed{\begin{aligned} \dot{x} &= f(x,t) \\ \dot{x}\Big|_{t+\frac{h}{2}} &= f\left(x\left(t+\tfrac{h}{2}\right), t+\tfrac{h}{2}\right) \end{aligned}}$$

2nd O RK Mid

$$x(t+h) = x(t) + hf\left(x\left(t+\frac{h}{2}\right), t+\frac{h}{2}\right) + O(h)^3$$

More Accurate to 3rd order terms

Euler was only to second!!

Small problem w/ this approach for some $\dot{x}$'s

(No problem if ~~only~~ $\dot{x}(t)$) (not x)

does happen often in Phys

We don't know $x(t+\frac{h}{2})$

We can use Euler Method to approx it!

Euler would give:

$$x(t+\tfrac{h}{2}) = x(t) + \frac{h}{2} f(x,t)$$

2nd Order Runge-Kutta method:

almost pseudocode

define

$$k_1 = h f(x,t)$$

$$k_2 = h f(x + \tfrac{k_1}{2}, t + \tfrac{h}{2})$$

$$x(t+h) = x(t) + k_2$$

Ex:

$$\mathring{x} = 2t \qquad \left(\overset{\text{exact}}{x = t^2}\right)$$

given $x_0 = 0$
at $t_0 = 0$

$$f = 2t$$

4th Order Runge-Kutta

accurate to $O(h)^4$ &

$$k_1 = h f(x, t)$$

$$k_2 = h f\left(x + \frac{k_1}{2}, t + \frac{h}{2}\right)$$

$$k_3 = h f\left(x + \frac{k_2}{2}, t + \frac{h}{2}\right)$$

$$k_4 = h f(x + k_3, t + h)$$

$$x(t+h) = x(t) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Statement for Student Volunteer to Read Before Students Take the Survey:

The purpose of the "Student Feedback Survey" is to reflect on our experiences and learning in this course. Significantly, this survey provides the instructor with greater insight into student learning so that the instructor can improve the course in the future. This is intended to be an opportunity for growth for both students and the instructor.

Consequently, it is important to take this survey with the seriousness that it deserves. Please take your time and answer questions fully with honesty, responsibility, and fairness.

This is an anonymous survey. It is important that you fill it out individually (that is, not in conference with others in the class) and the results of the survey will not be traced back to any one student either by the instructor or Saint Mary's College.

Now you may start the survey.