Exam 1

Physics 105, Thursday March 23

You may use a 3"x5" card of notes, one side. NO PHONES.

Present *clear* and *complete* answers.

Unjustified answers will earn no points. Any person who has taken this class should be able to understand what you did just by reading your solution. A diagram and a few words usually help. Start calculations with definitions (*e.g.* $\vec{v} \equiv \frac{d\vec{r}}{dt}$), facts (*e.g.* Newton's laws), or commonly used equations (*e.g.* constant acceleration equations).

Some answers require integrals.

You're expected to do simple integrals like $\int cz^n dz$, $\int ce^{kx}dx$, $\int c\ln(ky)dy$, $\int \frac{1}{(a+r)}dr$, $\int c\cos(k\theta)d\theta$, or $\int c\sin(k\phi)d\phi$.

If it's not simple, you don't need to do the integration. Instead, move all constants out of the integral, reasonably simplify all terms, specify limits, and clearly write the integral in one of two ways:

1. If the order of integration doesn't matter, group terms for each variable. For example,

$$H = kq\int_a^R (r^2 - b^2)^{-1/2}dr\int_{\pi/3}^{\pi} \sin^2\theta d\theta\int_0^{2\pi} \csc\phi d\phi$$

2. If order matters (a variable is related to another), make the order of integration clear. For example,

$$T = 5\pi\int_0^3 \left[\int_0^1 \left(\int_0^{1-y} \sqrt{x^2 - a^2}dx\right) ydy\right] z^2dz, \text{ or}$$
$$T = 5\pi\int_0^3 z^2dz\left[\int_0^1 ydy\left(\int_0^{1-y} \sqrt{x^2 - a^2}dx\right)\right]$$

Constants, equations

These equations will be:

D.1 and D.2
D.11 and D.12
D.28 D.29 D.34 and D.35
D.25 D.26 and D.27

1. What is an *inertial frame*? Provide an explanation that involves one (or more) of Newton's laws.

2. Starting from the forces on a particle, determine

   (a) the differential equation (d.e.) that governs the particle's motion

   (b) $x(t)$. If makes sense, then $y(t)$ and $z(t)$ also.

3. Determine $v(x)$ for a particle.

   Start from the one-dimensional d.e. for the motion. *Don't* solve for $x(t)$ and $v(t)$. Use the fact that $\frac{dv}{dx} = \frac{dv}{dt}\frac{dt}{dx}$.

4. The general form of $x(t)$ has constants that need to be determined. For example, $x(t) = A(1-e^{-t/\tau})+B$ has constants A and B.

   (a) What information do you need to determine the constants?
   (b) You're given this information. Determine the constants.

5. What is *terminal velocity*? When it makes sense, determine the terminal velocity of a particle.

6. Show that conservation of momentum follows from applying Newton's second and third laws to a system with no net external force.

   (This is in your notes, not our text.)

7. When is it valid to use conservation of linear momentum? conservation of angular momentum? conservation of energy?

8. (a) Determine if a force is conservative or non-conservative. Justify your answer with a calculation. If it makes sense, determine $U(\vec{r})$.

   (b) Alternatively, determine the force associated with a given $U(\vec{r})$.

9. Derive the following expression. Start from the conservation of mechanical energy.

$$dt = \frac{dx}{\sqrt{\frac{2}{m}(E - U(x))}}$$

   The above is for a particle of mass m moving in one-dimension. How can the above expression be used?

10. From a graph of $U(x)$, determine

    equilibrium points, the stability of each equilibrium point, points about which a particle could oscillate, the direction and magnitude of the force at any point.

11. Determine if a particle will undergo simple harmonic motion.

    You will be given either $U(x)$ or the forces on the particle. Justify your answer with a calculation. State any approximations you use.

    If it makes sense, determine the period and frequency.

12. Derive the d.e. for simple harmonic motion. Start from Newton's 2nd law and a restoring force of $-kx$

    Show, through direct substitution and calculation, that this d.e. is satisfied by $x(t) = A\sin(\omega_0 t - \delta)$.

13. Show, through direct substitution and calculation, that each of the following equations satisfies the d.e. for damped oscillations[1]:

    (a) $x(t) = Ae^{-\beta t}\cos(\omega_1 t - \delta)$ when $\beta < \omega_0$ and $\omega_1^2 = \omega_0^2 - \beta^2$

    (b) $x(t) = (A + Bt)e^{-\beta t}$ when $\beta = \omega_0$

    (c) $x(t) = e^{-\beta t}(A_1 e^{\omega_2 t} + A_2 e^{-\omega_2 t})$ when $\beta > \omega_0$ and $\omega_2^2 = \beta^2 - \omega_0^2$

14. (a) Identify if a system is underdamped, critically damped, or overdamped.

    (b) What is ω_0? ω_1? Give a physical explanation.

    (c) What is β? What is $1/\beta$? Give a physical explanation. What are their units?

15. For an underdamped harmonic oscillator, calculate

    (a) its decay time, frequency and period
    (b) how much its amplitude has decayed after a time interval or after a number of oscillations
    (c) the damping parameter based on the values of the damped frequency and the characteristic (natural, undamped) frequency
    (d) some variation of the above

16. The sinusoidally driven oscillator has transient and steady state solutions.

    What is the transient solution? What is the steady state solution? How are the behaviors of two solutions physically different?

17. A driven, damped oscillator has known damping parameter, natural frequency, and driving force. Determine the amplitude and frequency of its steady state motion.

    Alternatively, you want a specific frequency and amplitude for the motion. Determine the driving force and the damping parameter needed.

18. (a) What is resonance? Give a physical explanation.

    (b) The particular solution to the sinusoidally driven oscillator[2] is

$$x(t) = \frac{A}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\omega^2\beta^2}}\cos(\omega t - \delta)$$

    Show that resonance frequency is $\sqrt{\omega_0^2 - 2\beta^2}$. Explain the steps in your calculation.

19. What is the quality factor, Q? How does it affect the resonance curve? What does it say about damping?

    Calculate Q for an oscillator. There are 3 ways.

[1] for a resistive force that depends linearly on velocity, $-bv$
[2] again, with a resistive force of $-bv$