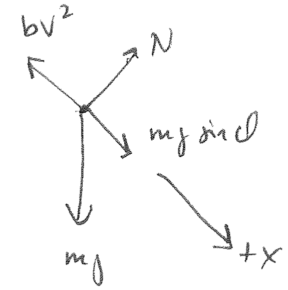
Exam 1: Solutions

1/



$$F_x = m\ddot{x}$$

$$mg\sin\theta - bv^2 = m\dot{v}$$

- terminal velocity is when resistive force becomes as large as the driving force (here, gravity), so the object no longer accelerates $\dot{v} = 0$ at $v = V_{terminal}$

$$mg\sin\theta - bv_t^2 = 0$$

$$V_t = \sqrt{\frac{mg}{b}\sin\theta}$$

2/

$$F_x = m\ddot{x}$$

$$-bv^{3/2} = m\dot{v}$$

here, $\dot{v} = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v$

$$-bv^{3/2} = m\frac{dv}{dx}v$$

$$\int_0^{X_f} dx = \int_{V_0}^{V_f} -\frac{m}{b}\frac{dv}{\sqrt{V}}$$

$$X_f = \left(-\frac{m}{b}\right)(2)\,v^{1/2}\Big|_{V_0}^{V_f}$$

$$X_f = -\frac{2m}{b}\left(\sqrt{V_f} - \sqrt{V_0}\right)$$

if $V_f = 0$

$$X_f = \frac{2m}{b}\sqrt{V_0}$$

3/ • Consider a 3 particle system of internal forces $\vec{F}_{12}, \vec{F}_{13}, \vec{F}_{21}, \vec{F}_{23}, \vec{F}_{31}, \vec{F}_{32}$.
and external forces $\vec{F}_{1ext}, \vec{F}_{2ext}, \vec{F}_{3ext}$

• the net force on the system is

$$\vec{F}_{net} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{21} + \vec{F}_{23} + \vec{F}_{31} + \vec{F}_{32} + \vec{F}_{1ext} + \vec{F}_{2ext} + \vec{F}_{3ext} = \frac{d\vec{p}_T}{dt}$$

where $\vec{F}_{net} = \frac{d\vec{p}_T}{dt}$ by Newton's 2nd law

• By Newton's third law, $\vec{F}_{12} = -\vec{F}_{21}$, $\vec{F}_{23} = \vec{F}_{32}$ & $\vec{F}_{31} = -\vec{F}_{13}$. These terms cancel.

$$\vec{F}_{net} = \vec{F}_{1ext} + \vec{F}_{2ext} + \vec{F}_{3ext} = \frac{d\vec{p}_T}{dt}.$$

• In the case of no net external force, $\frac{d\vec{p}_T}{dt} = 0$.

That is, momentum doesn't change over time... it's conserved.

4/ $$\vec{F} = -\vec{\nabla}U = -\left(\frac{\partial U}{\partial x}\hat{x} + \frac{\partial U}{\partial y}\hat{y} + \frac{\partial U}{\partial z}\hat{z}\right)$$

$$F_x = (-4\epsilon)\left[\sigma^{12}(-12)x^{-13} - \sigma^6(-6)x^{-7}\right]\hat{x}$$

$$F_x = -4\epsilon\left[-\frac{12\sigma^{12}}{x^{13}} + \frac{6\sigma^6}{x^7}\right]\hat{x}$$

equlbm is $F_x = 0$ $\quad \frac{-12\sigma^{12}}{x^{13}} + \frac{6\sigma^6}{x^7} = 0$

$$\frac{12\sigma^{12}}{x^{13}} = \frac{6\sigma^6}{x^7}$$

$$2\sigma^6 = x^6$$

$$2^{1/6}\sigma = x$$

5/ SHO is characterized by $U = \frac{1}{2}kx^2$ or $F = -kx$

$$U = -\frac{GMm}{R\sqrt{1+\frac{z^2}{R^2}}}$$

my mistake I dropped the − !

for small displacements $z \ll R$, $\left[1+\left(\frac{z}{R}\right)^2\right]^{-1/2} \approx 1 + \left(-\frac{1}{2}\right)\left(\frac{z^2}{R^2}\right)$

$$U = -\frac{GMm}{R}\left[1 - \frac{1}{2}\left(\frac{z^2}{R^2}\right)\right]$$

$$U = \frac{GMm}{R} + \underbrace{\frac{1}{2}\frac{GMm\,z^2}{R^3}}_{\frac{1}{2}kz^2}$$

where $k = \frac{GMm}{R^3}$

So $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{GM}{R^3}}$

6/ $F = m\ddot{x}$

$$-kx + F_0 \cos(\omega t) = m\ddot{x}$$

$$\boxed{\ddot{x} + \omega_0^2 x = \frac{F_0}{m}\cos(\omega t) \quad \text{when } \omega_0 = \sqrt{\frac{k}{m}}}$$

try $x(t) = D\cos\omega t$

$\ddot{x} = -\omega^2 D \cos\omega t$

$$-\omega^2 D\cos(\omega t) + \omega_0^2 D\cos(\omega t) = \frac{F_0}{m}\cos(\omega t)$$

$$(\omega_0^2 - \omega^2) D = F_0$$

$$D = \frac{F_0/m}{\omega_0^2 - \omega^2}$$

satisfies D.E. if this is true

7/ (a) $\omega^2 = \omega_0^2 - \beta^2$

so $\beta = \sqrt{\omega_0^2 - \omega^2} = \sqrt{\left(\frac{2\pi}{1}\right)^2 - \left(\frac{2\pi}{1.005}\right)^2} = 0.63$

(b) $A(t) = A_0 e^{-\beta t}$ $\quad t = 10 \times T = \frac{10 \cdot 2\pi}{\omega}$

$$\frac{A(t)}{A_0} = e^{-(0.6)(10 \cdot 1.005)}$$

$= 0.002$ down to 0.2% of original amplitude

8/ (a) critically damped, driven

(b) under damped

(c) under damped, driven $(\omega_1 < \omega)$ high frequency driving force

(d) " " $(\omega_1 > \omega)$

(e) free oscillator

(f) under damped

9/ $Q = \dfrac{\omega_R}{2\beta}$ in case of low damping $\omega_R \simeq \omega_0$ (act. calc shows $\omega_0 = 6.28$, $\omega_R = 6.22$)

$$\approx \frac{\frac{2\pi}{1}}{2(0.6)} = 5.2$$

Q's can be calculated for any system. It predicts behavior in the event that the system <u>is</u> driven: you'll be able to tell how well the system <u>selects</u> the resonance frequency from all the other frequencies.

10/ Yes, it does. $D = \dfrac{F_0/m}{\omega_0^2 - \omega^2}$ says that the resulting amplitude, D, depends on the driving frequency, ω. The amplitude peaks (is undefined!) at $\omega = \omega_0$. This peak in the response is the resonance phenomenon.