Exam 2

Physics 105, Tuesday April 25

You may use a 3"x5" card of notes, one side. NO PHONES.

Present *clear* and *complete* answers.

Unjustified answers will earn no points. Any person who has taken this class should be able to understand what you did just by reading your solution. A diagram and a few words usually help. Start calculations with definitions (*e.g.* $\vec{v} \equiv \frac{d\vec{r}}{dt}$), facts (*e.g.* Newton's laws), or commonly used equations (*e.g.* constant acceleration equations).

Some answers require integrals.

You're expected to do simple integrals like $\int cz^n dz$, $\int ce^{kx} dx$, $\int c\ln(ky)dy$, $\int \frac{1}{(a+r)}dr$, $\int c\cos(k\theta)d\theta$, or $\int c\sin(k\phi)d\phi$.

If it's not simple, you don't do the integration. Instead, move all constants out of the integral, reasonably simplify all terms, specify limits, and clearly write the integral in one of two ways:

1. If the order of integration doesn't matter, group terms for each variable. For example,

$$H = kq\int_a^R (r^2 - b^2)^{-1/2} dr \int_{\pi/3}^{\pi} \sin^2\theta d\theta \int_0^{2\pi} \csc\phi d\phi$$

2. If order matters, make the order of integration clear. For example,

$$T = 5\pi \int_0^3 z^2 dz \left[\int_0^1 y dy \left(\int_0^{1-y} \sqrt{x^2 - a^2} dx\right)\right]$$

Constants, equations

New material for this exam

Fourier series, calculus of variations, Lagrangian dynamics

1. Determine if a function $f(t)$ is anti-symmetric, symmetric, or neither.

   Which kind results in $\int_{-t_1}^{+t_1} f(t)dt = 0$?

2. Determine the Fourier series representation for a periodic function. This means

   (a) determining the integrals for a_n and b_n
   (b) identifying if either a_n or b_n are zero
   (c) writing the first few non-zero terms of the series. The terms should have numerical values. The only variable will be the time t.

3. A damped harmonic oscillator is driven by a periodic force $F(t)$. The Fourier representation of $F(t)$ is given. What is the steady state solution for the motion of the oscillator, $x_p(t)$? Write the

   (a) general solution as a summation
   (b) first few non-zero terms of $x_p(t)$.

4. (a) Write the differential arc length, ds, in Cartesian (x, y, z), cylindrical (ρ, ϕ, z), and spherical (r, θ, ϕ) coordinates.

   (b) Derive ds in cylindrical coordinates starting from $x = \rho\cos\phi$, $y = \rho\sin\phi$, $z = z$, and $ds = \sqrt{dx^2 + dy^2 + dz^2}$

5. Use the calculus of variations to determine the

   (a) geodesic for a surface (flat plane, cylinder, ...)
   (b) shortest time path between two points
   (c) function that minimizes a given integral.

6. What is Hamilton's principle? Use text and equations in your answer.

7. Show that the Lagrangrian and Newtonian formulation of mechanics are equivalent.

   Start with the Euler-Lagrange equations for a single particle. In addition to calculations, include text to explain your reasoning.

8. Write the Lagrangian for a system.

   Determine the equations of motion. (We're looking for the d.e. Simplify this in the form $\ddot{q}_1 + c\dot{q}_1 \ldots = \ldots$)

9. What are generalized coordinates? How many generalized coordinates do you need to describe a system?

   ~~What are generalized momenta? How are they calculated and how many are there for a given system?~~

10. (a) Write the speed squared, v^2, in Cartesian (x, y, z), cylindrical (ρ, ϕ, z), and spherical (r, θ, ϕ) coordinates.

    (b) Derive v^2 in cylindrical coordinates starting from $x = \rho\cos\phi$, $y = \rho\sin\phi$, $z = z$, and $v^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2$.

This exam will not cover everything we do in Lagrangian dynamics. The final exam will have more Lagrangian-related questions.

Exam 1 revisited

You can get up to 75% of your points back. For example, if you got 60% on exam 1, your new score can be as high as 90% ($= 100 - (40 \times 0.75)$).

Calculating v(x), determining SH motion, checking the solution to a d.e., identifying different SHO behavior

1. Determine $v(x)$ for a particle.

   Start from the one-dimensional d.e. for the motion. Don't solve for $x(t)$ and $v(t)$. Use the fact that $\frac{dv}{dx} = \frac{dv}{dt}\frac{dt}{dx}$.

   If it's possible, determine any constants of integration.

2. Determine if a particle will undergo simple harmonic motion.

   You will be given either $U(x)$ or the forces on the particle. Justify your answer with a calculation. State any approximations you use.

   If it makes sense, determine the period and frequency.

3. Show, through direct substitution and calculation, that a given $x(t)$ satisfies the d.e. for the motion of a particle. It may mean that there are constraints to the solution... What are they?

4. For a simple harmonic oscillator

   (a) Identify if it is driven, damped, or free.
   (b) If it's damped, identify if it is underdamped, critically damped, or overdamped.
   (c) What are ω, ω_0, ω_1, ω_2, and ω_R? Give physical explanations (not equations!).
   (d) What is a transient solution? steady state solution? How are the behaviors of the two solutions physically different?