Final Exam

Physics 105, Tuesday May 23 1-3pm

You may use an 8.5"x11" sheet of notes, one side. NO PHONES.

Present *clear* and *complete* answers.

Unjustified answers will earn no points. Any person who has taken this class should be able to understand what you did just by reading your solution. A diagram and a few words usually help. Start calculations with definitions (*e.g.* $\vec{v} \equiv \frac{d\vec{r}}{dt}$), facts (*e.g.* Newton's laws), or commonly used equations (*e.g.* constant acceleration equations).

Some answers require integrals.

You're expected to do simple integrals like $\int cz^n dz$, $\int ce^{kx} dx$, $\int c\ln(ky)dy$, $\int \frac{1}{(a+r)}dr$, $\int c\cos(k\theta)d\theta$, or $\int c\sin(k\phi)d\phi$.

If it's not simple, you don't do the integration. Instead, move all constants out of the integral, reasonably simplify all terms, specify limits, and clearly write the integral in one of two ways:

1. If the order of integration doesn't matter, group terms for each variable. For example,

$$H = kq\int_a^R (r^2 - b^2)^{-1/2} dr \int_{\pi/3}^{\pi} \sin^2\theta d\theta \int_0^{2\pi} \csc\phi d\phi$$

2. If order matters, make the order of integration clear. For example,

$$T = 5\pi \int_0^3 z^2 dz \left[\int_0^1 y dy \left(\int_0^{1-y} \sqrt{x^2 - a^2} dx\right)\right]$$

Revised course grade weights

The final grade will be based on

Problem sets	~~25%~~ *27%*
Project	~~10%~~ *13%*
Midterm exams	35%
Final exam	~~30%~~ *25%*

New material for this exam

1. (a) Write the differential arc length, ds, in Cartesian (x, y, z), cylindrical (ρ, ϕ, z), and spherical (r, θ, ϕ) coordinates.

   (b) Derive ds in cylindrical coordinates starting from $x = \rho\cos\phi$, $y = \rho\sin\phi$, $z = z$, and $ds = \sqrt{dx^2 + dy^2 + dz^2}$

   (c) Write the speed squared, v^2, in Cartesian (x, y, z), cylindrical (ρ, ϕ, z), and spherical (r, θ, ϕ) coordinates.

   (d) Derive v^2 in cylindrical coordinates starting from $x = \rho\cos\phi$, $y = \rho\sin\phi$, $z = z$, and $v^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2$.

2. Show that the Lagrangrian and Newtonian formulation of mechanics are equivalent.

   Start with the Euler-Lagrange equations for a single particle. In addition to calculations, include text to explain your reasoning. *Use your own words.*

3. Show that if the Lagrangian is invariant with respect to translations in space in a certain direction, then the total linear momentum in that direction is conserved.

   In addition to calculations, include text to explain your reasoning. *Use your own words.*

4. What are generalized momenta? What are generalized forces? How are they calculated and how many are there for a given system?

5. Write the Hamiltonian for a system.

   Determine Hamilton's equations of motion for the system.

6. Determine/show whether the Hamiltonian is equal to the total energy of the system.

   Determine whether the Hamiltonian is conserved.

From before

1. Show, through direct substitution and calculation, that a given $x(t)$ satisfies the d.e. for the motion of a particle.

   It may mean that there are constraints to the solution... What are they?

2. From $U(x)$ determine equilibrium points, the stability of each point, and the direction and magnitude of the force at any point.

   For a given force $\vec{F}$, determine if it's conservative or nonconservative. If it's conservative, determine the potential energy associated with it.

3. Determine if a particle will undergo simple harmonic motion.

   You will be given either $U(x)$, the forces on the particle, or the equation of motion (d.e. for a coordinate). Justify your answer with a calculation. State any approximations you use.

   If it makes sense, determine the period and frequency.

4. Determine the Fourier series representation for a periodic function. This means

   (a) determining the integrals for a_n and b_n
   (b) identifying if either a_n or b_n are zero
   (c) writing the first few non-zero terms of the series. The terms should have numerical values. The only variable will be the time t.

5. Use the calculus of variations to determine the function that minimizes a given integral.

6. Write the Lagrangian for a system.

   Determine the equations of motion. (We're looking for the d.e. Simplify this in the form $\ddot{q}_1 + c\dot{q}_1 \ldots = \ldots$)