Brachistochone. What path allows a particle to travel from A to B in the shortest amount of time? *Brachistos* = short, *chronos* = time.

1. Start with writing the integral for total time T

$$T = \int dt$$

Substitute in the following, in some coherent manner

$v = ds/dt$, $ds = \sqrt{dx^2 + dy^2}$, and an equation $v(y)$ (conservation of energy $mgy = \frac{1}{2}mv^2$)

2. Rewrite your integral in the form

$$T = \int f(x, x'; y) dy$$

where $x' = dx/dy$.

We switched from $f(y, y'; x)$ to $f(x, x'; y)$. It's ok. It results in $x(y)$ instead of $y(x)$. The integral is easier this way. It also means that instead of $\frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'} = 0$, we'll be using $\frac{\partial f}{\partial x} - \frac{d}{dy}\frac{\partial f}{\partial x'} = 0$

What is $f(x, x'; y)$?

3. Use Euler's equation and simplify. You should find the following:

$$x' = \sqrt{\frac{y}{2a - y}}$$

4. Write the integral that will give you $x(y)$.

Soap film. Soap films take on shapes that minimize the surface area (of course it also depends on what the film hangs on... those are boundary conditions). Consider points $P1 = (x_1, y_1)$ and $P2 = (x_2, y_2)$. What path between these points, when rotated around the y-axis will lead to a minimum in surface area?

1. Write the integral for area A

$$A = \int dA$$

Use as your dA a thin strip of width ds. If you unfold this strip to make it rectangular-ish, what is its area dA?

2. Substitute in $ds = \sqrt{dx^2 + dy^2}$, move out any constants, and rewrite your integral in the form

$$A = \int f(y, y'; x) dx$$

What is $f(y, y'; x)$?

3. Apply Euler's equation and simplify. You should find the following:

$$y' = \frac{c}{\sqrt{x^2 - c^2}}$$

where c is a constant. Write the integral you'd need to find $y(x)$

4. This integral is not uncommon and has a neat solution. Look it up. See appendix E.1 in your text. Also look up an identity for cosh in terms of the natural log ln. Write both of these equations.

5. The solution is $y = c \cosh^{-1}(\frac{x}{c})$. Plot this. Use Wolfram alpha or your favorite fast graphing program. Try $c = 5$, $b = 8$. What do the 5 and 8 represent?

Fourier series: sawtooth wave

Consider the following sawtooth wave,

$$F(t) = At/\tau = \omega At/2\pi, \qquad -\tau/2 < t < \tau/2$$

Represent this function as a Fourier series,

$$F(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}[a_n \cos(n\omega t) + b_n \sin(n\omega t)]$$

where the coefficients are

$$a_n = \frac{2}{\tau}\int_0^{\tau} F(t') \cos(n\omega t')dt' = \frac{\omega}{\pi}\int_{-\pi/\omega}^{+\pi/\omega} F(t') \cos(n\omega t')dt'$$

$$b_n = \frac{2}{\tau}\int_0^{\tau} F(t') \sin(n\omega t')dt' = \frac{\omega}{\pi}\int_{-\pi/\omega}^{+\pi/\omega} F(t') \sin(n\omega t')dt'$$

1. Is this sawtooth symmetric, anti-symmetric, or neither?

2. (a) Calculate a_n. Write the integral. Include limits.
(b) Calculate b_n. Write the integral. Include limits.

3. The coefficients a_n are zero. Why? Use symmetry in your explanation.

3. After integrating, $b_n = \frac{A}{n\pi}(-1)^{n+1}$. Write down the first three terms of the series.

Plot this series (use a fourier series applet).