In-class problems, Wed Feb 21, 2018

Ch2 projectiles and charged particles

1. The physical origin of quadratic drag is inertia of the fluid the projectile sweeps up. The projectile has a cross-sectional area A, speed v, and the fluid density is ρ

   Show that the small mass of water the projectile encounters in a small time is $dm = \rho A v dt$.

   (a) Write an expression for the fluid volume dV swept up in a time dt. Use dt and some of the given parameters.

   (b) Write dm in terms of dV and the density.

   This dm is accelerated to the speed of the projectile.

   (c) Write the momentum change of the fluid, dp. Use dm and the speed.

   (d) Determine the rate of change of momentum dp/dt. This is the force ($F = \dot{p}$).

   (e) Include a multiplicative factor of κ to account for the shape of the projectile.

   (adapted from Taylor 2.4. Final answer: $f_{\mathrm{quad}} = \kappa \rho A v^2$)

2. A projectile is subject to linear air resistance is thrown vertically down with speed $2v_{\mathrm{ter}}$.

   (a) Write the expression for $v(t)$ with this initial condition and plot it. Use results we already have.
   (b) How does this compare with the $v(t)$ for a projectile dropped from rest, also with linear air resistance.

   (adapted from Taylor 2.5)

3. A steel ball bearing is dropped in glycerin. In addition to a linear drag force, there is a bouyant force $F_b = \rho_{\mathrm{fluid}} g V$ on the object. The ball bearing has a diameter of 2mm and density of 7.8 g/m^3. Glycerin has a density of 1.3 g/cm^3 and a viscosity of 12 Ns/m^2 at STP.

   (a) Determine an expression and a value for the terminal velocity of the steel ball bearing. When in doubt, start with Newton's second law.
   (b) How does this change the solution for $v(t)$? Determine and calculate the characteristic time τ.

   (Taylor 2.10. Answers: $v_{\mathrm{ter}} = mg(1 - \rho_{\mathrm{fluid}}/\rho_{\mathrm{steel}})/b$, 1.2 mm/s, 1.2 ms.)

4. A sphere (ρ_{sph}, diameter D) falls through air (ρ_{air}), with a purely quadratic drag force.

   (a) Use the result that $f_{\mathrm{quad}} = \frac{1}{4}\rho_{\mathrm{air}} A v^2$ to derive the following expression for the terminal speed

   $$v_{\mathrm{ter}} = \sqrt{\frac{8}{3} D g \frac{\rho_{\mathrm{sph}}}{\rho_{\mathrm{air}}}}$$

   If two spheres have the same (b) diameter but different densities (material), will a denser one fall faster or slower?
   (c) densities (material) but different diameters, will the larger one fall faster or slower?

5. 1D motion with linear drag has the solutions

   $$v = v_0 e^{-t/\tau}$$

   $$x(t) = v_0 \tau (1 - e^{-t/\tau})$$

   Suppose we'd like to know the speed as a function of position, $v(x)$. Yes, you can solve for t and substitute. But here's a more elegant way.

   (a) Write Newton's second law for this situation. Include only a linear drag force, and let $\ddot{r} = \dot{v}$.

   (b) Note that $\dot{v} = dv/dt$. This will naturally lead to $v(t)$. We want $v(x)$, so note that

   $$\frac{dv}{dx} = \frac{dv}{dt}\frac{dt}{dx} = \frac{dv}{dt} \cdot \frac{1}{v}$$

   Rewrite the above note to get an expression for $\frac{dv}{dx}$ in terms of $\frac{dv}{dt}$.

   (c) Sub your result from (b) into Newton's second law. You should now have an differential equation with dv and dx, with no dt

   (d) Solve your differential equation to find $v(x)$. Initial conditions: at $t = 0$, $x = 0$ and $v = v_0$.

   (adapted from Marion Thornton Ex 2.4. Answer: $v = v_0 - x/\tau$)