In-class problems, Mon Mar 5, 2018

4.1 4.2 Kinetic energy, potential energy, work-energy theorem, conservation of energy

1. Derive the work-energy theorem.
   (a) Start with the definition of kinetic energy $T = \frac{1}{2}mv^2$. Substitute in $v^2 = \vec{v} \cdot \vec{v}$.
   (b) Take the derivative wrt time of T: $dT/dt = \ldots$.
   Remember to use the product rule, then collect terms. (answer for this: $m\dot{\vec{v}} \cdot \vec{v}$)
   (c) Identify the term that's the net force $\vec{F}_{\text{net}}$. Sub this in. Also sub in $\vec{v} \equiv d\vec{r}/dt$
   (d) Both sides of the equation are change with respect to time. That is *something*$/dt$. It's the same dt on both sides, so "cancel" them.
   (e) Write the final expression... $dT = \ldots$

2. Consider the force $\vec{F} = \vec{r}$. A particle move along the path shown below. Calculate the work done by this force.
   (a) Write $W = \int \vec{F} \cdot d\vec{r}$. Sub in the expression for $\vec{F}$.
   (b) Sub in $\vec{r} = x\hat{\imath} + y\hat{\jmath}$. Sub in $d\vec{r} = dx\hat{\imath} + dy\hat{\jmath}$. Carry through the dot product.
   (c) Let's change everything to y and dy. Sub in $x = 1 - y^2$ and $dx = -2ydy$. Put in the limits for y.
   (d) Evaluate your integral (answer: 0).

3. A charged particle q is in uniform electric field $\vec{E} = E_0\hat{\imath}$. It moves from (0,0) to (x, y). Determine the potential energy, $U(\vec{r})$.
   (a) Write $W = \int \vec{F} \cdot d\vec{r}$. Sub in the expression for $\vec{F} = q\vec{E}$, and for $\vec{E}$.
   (b) Sub in $d\vec{r} = dx\hat{\imath} + dy\hat{\jmath}$.
   (c) Choose your path. Any path will do, since this is a conservative force. So choose the easiest path and simplify your integral.
   (d) Integrate and find $U(\vec{r})$

4. Derive conservation of mechanical energy.
   (a) Write down the work energy theorem, $\Delta T = \int_1^2 \vec{F}(\vec{r}) \cdot d\vec{r}$.
   (b) Now substitute in the definition of work, $W =$. Use the expression that has an integral and a force.
   (c) Now substitute in the relationship between work and potential energy for conservative forces $W(1 \rightarrow 2) =$ something with ΔU.
   (d) Move terms to one side, and use the definition $E_{\text{mech}} = T + U$.

Due Wed Mar 7, 2018

Revisit your text and notes for Ch 1-3, this part of 4

1. Identify one problem or derivation that you'd like to revisit.
2. Repeat
3. Repeat