Exam 1

Physics 105, Monday March 23 2018

You may use a 3"x5" card of notes, one side. NO PHONES, no calculators.

Present *clear* and *complete* answers.

Unjustified answers will earn no points. Any person who has taken this class should be able to understand what you did just by reading your solution. A diagram and a few words usually help. Start calculations with definitions (*e.g.* $\vec{v} \equiv \frac{d\vec{r}}{dt}$), facts (*e.g.* Newton's laws), or commonly used equations (*e.g.* constant acceleration equations).

Do simple integrals. Leave complex ones unevaluated.

You're expected to do integrals like $\int cz^n dz$, $\int ce^{kx}dx$, $\int c\ln(ky)dy$, $\int \frac{1}{(a+r)}dr$, $\int c\cos(k\theta)d\theta$, or $\int c\sin(k\phi)d\phi$. And anything from the table at front of our text:

Some Integrals

$$\int \frac{dx}{1+x^2} = \arctan x \qquad \int \frac{dx}{1-x^2} = \operatorname{arctanh} x$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x \qquad \int \frac{dx}{\sqrt{1+x^2}} = \operatorname{arcsinh} x$$

$$\int \tan x\, dx = -\ln\cos x \qquad \int \tanh x\, dx = \ln\cosh x$$

$$\int \frac{dx}{x+x^2} = \ln\left(\frac{x}{1+x}\right) \qquad \int \frac{x\, dx}{1+x^2} = \ln(1+x^2)$$

$$\int \frac{dx}{\sqrt{x^2-1}} = \operatorname{arccosh} x \qquad \int \frac{x\, dx}{\sqrt{1+x^2}} = \sqrt{1+x^2}$$

$$\int \frac{dx}{x\sqrt{x^2-1}} = \arccos(1/x) \qquad \int \frac{\sqrt{x}\, dx}{\sqrt{1-x}} = \arcsin(\sqrt{x}) - \sqrt{x(1-x)}$$

$$\int \frac{dx}{(1+x^2)^{3/2}} = \frac{x}{(1+x^2)^{1/2}} \qquad \int \ln(x)\, dx = x\ln(x) - x$$

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}\sqrt{1-mx^2}} = K(m), \quad \text{complete elliptic integral of first kind}$$

Don't do the integration on anything more complex. Instead, move all constants out of the integral, reasonably simplify all terms, specify limits, and clearly write the integral in one of two ways:

1. If the order of integration doesn't matter, group terms for each variable. For example,

$$H = kq\int_a^R (r^2-b^2)^{-1/2}dr \int_{\pi/3}^{\pi} \sin^2\theta d\theta \int_0^{2\pi} \csc\phi d\phi$$

2. If order matters (a variable is related to another), make the order of integration clear. For example,

$$T = 5\pi\int_0^3 z^2 dz \int_0^1 y dy \left(\int_0^{1-y} \sqrt{x^2-a^2}dx\right)$$

1. In intro physics, Newton's second law is presented as

$$\sum \vec{F} = m\vec{a}$$

What are other ways to write $m\vec{a}$?

2. We describe position with the vector $\vec{r}$. Using Cartesian coordinates, write

$$\vec{r}, d\vec{r}, \ddot{\vec{r}}, \text{and } \hat{\mathrm{r}}$$

3. In Cartesian, Newton's second law in 2D is

$$\vec{F} = m\ddot{x}\hat{x} + m\ddot{y}\hat{y}$$

(a) Write Newton's second law in polar coordinates. That is, use $\hat{r}$ and $\hat{\theta}$.

(b) The above isn't simply $\vec{F} = m\ddot{r}\hat{r}$. Why not? Don't to derive (a). Provide the main ideas of the derivation.

4. Apply Newton's second law to determine the d.e.(s) that governs the particle's motion. In general, the form should look something like $\ddot{s} = \ldots$.

Be able to

   (a) do this in 1D
   (b) include gravity, friction, tension, linear drag, quadratic drag, spring force, electrostatic force, ...
   (c) do this in 2D
   (d) determine if a set of equations (for example in x and y) are coupled or uncoupled
   (e) do this in a tilted frame (for example along an incline).

5. Solve some common differential equations. For example, $\ddot{x} =$ constant, $\dot{v} = -kv$, or $\dot{v} = a - bv$.

The general solution has constants of integration. For example, $x(t) = A(1 - e^{-t/\tau}) + B$. What information do you need to determine A and B? Say you're given this information. Determine A and B.

6. Determine $v(x)$ for a particle.

Start from the one-dimensional d.e. for the motion. Don't determine $x(t)$ and $v(t)$. Use the fact that $\frac{dv}{dx} = \frac{dv}{dt}\frac{dt}{dx}$.

7. Determine

(a) maximum heights or positions of a particle
(b) position or time that a particle reaches a particular velocity
(c) range for projectiles
(d) the trajectory of a particle $y(x)$

Address all the above with no air resistance.

Address (a) and (b) when including air resistance in 1D.

Qualititatively say something about (c) (d), when including air resistance in 2D motion. Why are we not handling this *quantitatively* at this point?

8. What's the physical origin of the linear drag force? What about the quadratic drag force?

When do you use linear? quadratic? both?

9. What is terminal velocity?

When it makes sense, determine the terminal velocity of a particle.

Often, $v(t)$ has a *characteristic time* or *time constant*. What does it represent? What happens after 1 time constant? 2 time constants? 3 time constants?

10. Determine the Taylor series expansion of a function.

Why do we often use the series expansion of a function instead of the actual function? It may be useful to give an example.

11. Show that conservation of momentum follows from applying Newton's second and third laws to a system with no net external force.

You'll probably be asked do this for a two or three particle system.

12. Apply conservation of momentum to explosions in 2D, car crashes in 2D, rockets in 1D. This usually means finding the initial or final velocities of a particle.

13. Calculate the center of mass of a set of point particles, or an extended object (continuous mass distribution).

14. Calculate the angular momentum of a single particle.

When is the angular momentum of a single object or system conserved?

15. Calculate the moment of inertia of a set of particles or an extended object.

16. Derive the work-kinetic energy theorem. Start with

$$\frac{dT}{dt} = \frac{d(\frac{1}{2}m\vec{v}\cdot\vec{v})}{dt}$$

and work down to $dT = \vec{F}\cdot d\vec{r}$. Explain your steps and assumptions.

What does the work-kinetic energy theorem say? Give an example.

17. Calculate the work done by a force on an object. Be sure to indicate if it's positive or negative.

Determine whether net work is done an object.

18. Determine if a force is conservative.

Determine the potential energy of a conservative force.

Determine the force associated with a potential energy.