Exam 2

Physics 105, Friday April 27 2018

You may use a 3"x5" card of notes, one side. NO PHONES, no calculators.

Present *clear* and *complete* answers.

Unjustified answers will earn no points. Any person who has taken this class should be able to understand what you did just by reading your solution. A diagram and a few words usually help. Start calculations with definitions (*e.g.* $\vec{v} \equiv \frac{d\vec{r}}{dt}$), facts (*e.g.* Newton's laws), or commonly used equations (*e.g.* constant acceleration equations).

Do simple integrals. Leave complex ones unevaluated.

You're expected to do integrals like $\int cz^n dz$, $\int ce^{kx} dx$, $\int c\ln(ky)dy$, $\int \frac{1}{(a+r)}dr$, $\int c\cos(k\theta)d\theta$, or $\int c\sin(k\phi)d\phi$. And anything from the table at front of our text:

Some Integrals

$$\int \frac{dx}{1+x^2} = \arctan x \qquad \int \frac{dx}{1-x^2} = \operatorname{arctanh} x$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x \qquad \int \frac{dx}{\sqrt{1+x^2}} = \operatorname{arcsinh} x$$

$$\int \tan x\, dx = -\ln\cos x \qquad \int \tanh x\, dx = \ln\cosh x$$

$$\int \frac{dx}{x+x^2} = \ln\left(\frac{x}{1+x}\right) \qquad \int \frac{x\, dx}{1+x^2} = \ln(1+x^2)$$

$$\int \frac{dx}{\sqrt{x^2-1}} = \operatorname{arccosh} x \qquad \int \frac{x\, dx}{\sqrt{1+x^2}} = \sqrt{1+x^2}$$

$$\int \frac{dx}{x\sqrt{x^2-1}} = \arccos(1/x) \qquad \int \frac{\sqrt{x}\, dx}{\sqrt{1-x}} = \arcsin(\sqrt{x}) - \sqrt{x(1-x)}$$

$$\int \frac{dx}{(1+x^2)^{3/2}} = \frac{x}{(1+x^2)^{1/2}} \qquad \int \ln(x)\, dx = x\ln(x) - x$$

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}\sqrt{1-mx^2}} = K(m), \quad \text{complete elliptic integral of first kind}$$

Don't do the integration on anything more complex. Instead, move all constants out of the integral, reasonably simplify all terms, specify limits, and clearly write the integral in one of two ways:

1. If the order of integration doesn't matter, group terms for each variable. For example,

$$H = kq\int_a^R (r^2-b^2)^{-1/2}dr \int_{\pi/3}^{\pi} \sin^2\theta d\theta \int_0^{2\pi} \csc\phi d\phi$$

2. If order matters (a variable is related to another), make the order of integration clear. For example,

$$T = 5\pi\int_0^3 z^2 dz \int_0^1 ydy \left(\int_0^{1-y} \sqrt{x^2-a^2}dx\right)$$

1. Is a time-dependent force $\vec{F}(\vec{r}, t)$ conservative? Explain.

2. From a graph or function of $U(x)$, determine

   equilibrium points, the stability of each equilibrium point, points about which a particle could oscillate, the direction and magnitude of the force at any point.

3. Determine if a particle will undergo simple harmonic motion.

   You'll be given either $U(x)$ or the forces on the particle. State any approximations you use.

   If it makes sense, determine the period and frequency.

4. There are 4 forms for the solution of a simple harmonic oscillator (undriven, undamped).

   What are these solutions?

   Show, by direct substitution, that each solution satisfies the d.e.

   Show that any two are equivalent to each other.

5. Write the equation of motion for an oscillator. That is, write $\ddot{x} = \ldots$.

   Start from Newton's 2nd law and a restoring force of $-kx$. You may be asked include damping and driving forces.

6. For a simple harmonic oscillator

   (a) Identify if it's driven, damped, or free.

   (b) If it's damped, identify if it is underdamped, critically damped, or overdamped.

   (c) What are ω, ω_0, ω_1, and ω_2 (also referred to as ω_R)? Give physical explanations as well as equations.

   (d) What is a transient solution? steady state solution? How are the behaviors of the two solutions physically different?

7. Consider a damped harmonic oscillator.

   (a) What is the general solution $x(t)$ for each case: underdamped, critically damped, and overdamped. Be able to find the constants of integration from intial conditions.

   In the underdamped case, calculate

   (b) its decay time, frequency and period
   (c) how its amplitude has decayed after a time interval or after a number of oscillations
   (d) the damping parameter based on the values of the damped frequency and the characteristic (natural, undamped) frequency
   (e) some variation of the above

8. Consider a driven oscillator.

   (a) What will be the oscillator's frequency, amplitude and phase? What does the phase represent?

   (b) What is resonance? Give a physical description of the phenomenon.

   (c) Derive the equation for the resonance frequency, $\omega_2 = \sqrt{\omega_0^2 - 2\beta^2}$. Start from $x_p(t)$ and the expression for the amplitude A. Give your reasoning for one or two steps.

9. (a) What does the quality factor Q describe in terms of the resonance curve?

   (b) What is the FWHM or HWWM? A diagram would be extremely helpful here.

   (c) Calculate Q for an oscillator. There are two ways.

10. Determine the Fourier series for a periodic function. This means

    (a) determining the integrals for a_0, a_n, and b_n

    (b) the coefficients a_n or b_n may be zero, depending on the even- or odd-ness $f(t)$. Determine whether a given $f(t)$ is even (symmetric), odd (antisymmetric), or neither.

    (c) writing the first few non-zero terms of the series. The terms should have numerical values. The only variable will be the time t.

11. A damped harmonic oscillator is driven by a periodic force $F(t)$. The Fourier representation of $F(t)$ is given. What is the steady state solution for the motion of the oscillator, $x_p(t)$? Write the

    (a) general solution as a summation
    (b) first few non-zero terms of $x_p(t)$.

12. (a) Write the differential arc length, ds, in Cartesian (x, y, z), cylindrical (ρ, ϕ, z), and spherical (r, θ, ϕ) coordinates.

    (b) Derive ds in cylindrical coordinates starting from $x = \rho\cos\phi$, $y = \rho\sin\phi$, $z = z$, and $ds = \sqrt{dx^2 + dy^2 + dz^2}$

13. Use the calculus of variations to determine the

    (a) geodesic for a surface (flat plane, cylinder, ...)
    (b) shortest time path between two points
    (c) function that minimizes a given integral.