Analytical mechanics: computational project

Choose a classical mechanics problem that has no analytical solution. Explore the motion of the system using computational methods. This project is an opportunity to go further into a problem than what we've done, or look at a real-world system. It's also a chance to use your computational skills, and gain more experience presenting scientific/technical information. Some ideas for projects:

> The not-so-simple harmonic oscillator. For example, in a pendulum we often use the small angle approximation, $\sin\theta \approx \theta$. What's the motion in the case that we *don't* use the approximation?
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> Two dimensional projectile motion with quadratic drag. What do the trajectories, range or maximum height look like?
>
> One dimensional motion with varying drag. For example, what's the motion of an object falling through Earth's atmosphere, given that the drag parameter depends on the atmospheric density, $c = Ae^{-y/H}$?
>
> The simple-harmonic oscillator with quadratic drag.

Structure for the presentation and report

State the problem. Describe the system. Use text and at least one diagram. Bonus points if you motivate your problem: is this a real-world system? is there a specific application? why is this interesting?

Determine the equations of motion. Explain the logic behind important steps. Define each variable; use a diagram. It can be the same diagram from the problem statement, but make sure to include labels (distances, angles, etc that you use).

Discuss the equations. For example, does it recover simple behavior (analytically solvable behavior) in certain limits? What are those limits and what do you expect to happen? Are there equilibrium points?

Discuss your computational method, and demonstrate that your code works. We'll be using Euler's method. Include 1 paragraph or slide that shows how this works for your system. Then do two things to demonstrate that your code works:

1. Choose a set of parameters and initial conditions. Hand calculate the first 3 values. Run your code and show that these first 3 values match your hand calculated values.
2. Choose a set of parameters and initial conditions that leads to simple, predictable behavior. Run your code and show (via a graph or animation) that it leads to this expected behavior.

Present your findings for non-analytical solutions.
Explore the motion of your system and report one set of results. While $x(t)$ or $v(t)$ always works, other quantities may be more useful. For example, if your system oscillates, look at periods, frequencies, amplitudes, or phase shifts. If you're looking a projectile, look at range, max height, or trajectory. Energy is a good thing to keep track of for any motion, possibly momentum. Your results will consist of one of the following:

- Vary a parameter systematically, while keeping initial conditions fixed. For example, change the mass from small to large. Keep the initial position, initial velocity and other parameters (like length) the same. See how the behavior evolves.
- Vary an initial condition systematically, while keeping parameters fixed. For example, change the initial velocity from small to large (and from positive to negative). Keep other initial conditions and all parameters fixed. See how the behavior evolves.
- Investigate how the behavior evolves from (or goes to) a known behavior. For example, going from simple harmonic oscillation to something *not* harmonic. Or if a system oscillates around a local minimum, at some point it will probably escape that minimum.

Dates and format of presentation and report

Project idea proposal: Mon Apr 30
Submit 1-2 sentences and a diagram about what you've chosen to do. Describe the system and what you plan to explore.

In class presentations: Mon May 14, Wed May 16
8 minutes. You can present all of it in a presentation application (Powerpoint/Google Slides/Prezi), or you can use the board for part of it (description of problem, equations of motion). The "it's working" demonstration should be done live (Matlab). Show your results in some kind of presentation app or Matlab.

Written report: Thu May 24
Use LaTeXor any document application that allows for equations. Email me a pdf. Two pages single spaced text, plus additional space for graphs.