Physics 110 Fall 2018

9/6 Tutorial 1–Second derivatives

The gradient, divergence, and curl are all the ways you can take "first derivatives" with the ∇ operator. Let's check out second derivatives.

For all the problems on this sheet, consider f and g are scalar functions, and $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{v}$ are vector functions. All could be functions of x, y, and z.

For all the problems on this sheet, use

$$\vec{v}_a = x^2\hat{x} + 3xz^2\hat{y} - 2xz\hat{z}$$

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1. **The divergence of a gradient**
   Write out all the components of
   $$\nabla \cdot (\nabla f)$$
   The divergence of a gradient is also called the **Laplacian**, and it will be obvious why it's written $\nabla^2 f$.

2. Find the Laplacian (∇^2) of $f = \sin(x)\sin(y)\sin(z)$

3. **The curl of a gradient**
   Write out all the components of
   $$\nabla \times (\nabla f)$$
   and simplify.

4. You should have found that the curl of a gradient is always zero. Let's test your intuition on two sort of similar examples.

   (a) Check $(\vec{A}f) \times (\vec{A}g)$.

   (b) Check $(\nabla f) \times (\nabla g)$

   The point of this exercise is to show that the curl of a gradient is zero not because of something like (a). The derivatives make it different.

5. **The divergence of a curl**
   Show that
   $$\nabla \cdot (\nabla \times \vec{v})$$
   is zero.

   Again, derivatives make things different. You canNOT do this problem by using the vector identity:
   $$\vec{A} \cdot (\vec{B} \times \vec{C}) = (\vec{A} \times \vec{B}) \cdot \vec{C})$$
   You will have to just write out components again.

6. Test it on vector $\vec{v}_a$. (This is the same as asking: find the divergence of the curl of $\vec{v}_a$.)