# 10/30–Tutorial on Ampere's Law

This begins where we left off with the previous InClass Worksheet. Several of these are from CU Boulder, and several from Griffiths. With some of my own.

## Tutorial on Ampere's Law

Ampere's Law in integral form is:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$$

Learning how to choose an Amperian Loop is a skill we should practice.

1. Imagine there is a constant magnetic field $B$ whose direction is given by the field lines shown below. An Amperian loop is also shown below (dashed lines).

   (a) What is $\int \vec{B} \cdot d\vec{\ell}$ for each side of the loop?

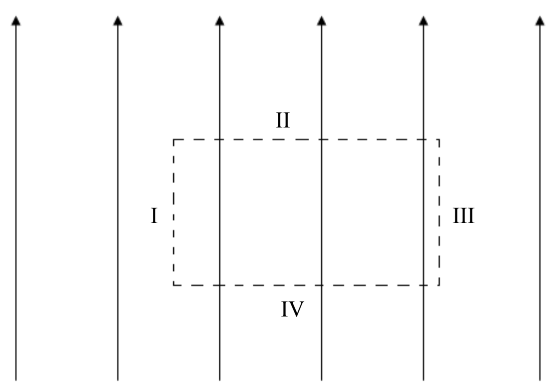
      i. Side I:

      ii. Side II:

      iii. Side III:

      iv. Side IV:

   (b) What is $\oint \vec{B} \cdot d\vec{\ell}$ ?




2. Now imagine rotating the Amperian loop such that it makes an angle $\theta$ with respect to the magnetic field (shown below).

   (a) What is $\int \vec{B} \cdot d\vec{\ell}$ for each side of the loop?

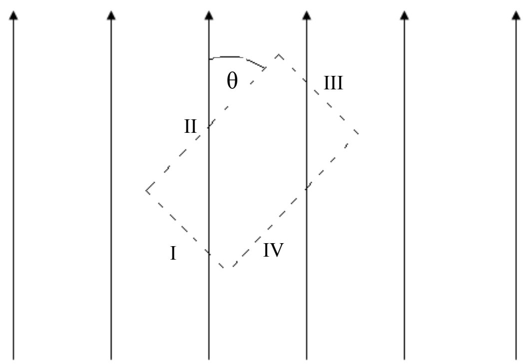
      i. Side I:

      ii. Side II:

      iii. Side III:

      iv. Side IV:

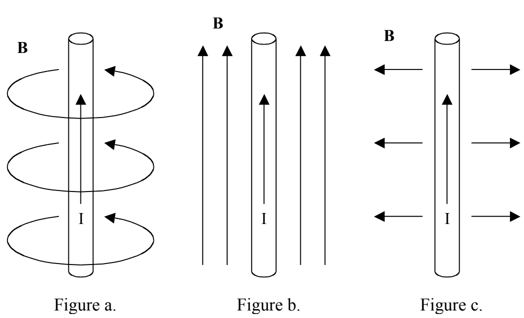
   (b) What is $\oint \vec{B} \cdot d\vec{\ell}$ ?




3. Compare $\oint \vec{B} \cdot d\vec{\ell}$ for the two previous cases. Do they make sense? Explain.
4. Thinking about Problems 1 and 2:
   (a) Qualitatively explain how your results for questions 1 and 2 would change if your Amperian loop was a circle instead of a rectangle.
   (b) Why is a rectangular Amperian loop better for this problem than a circular Amperian loop? Explain.
   (c) What sort of situation might you want a circular Amperian loop for and why? Be explicit.
5. If for an Amperian loop, $\oint \vec{B} \cdot d\vec{\ell} = 0$ (not necessarily the one in questions 1 and 2), can you conclude anything about the magnetic field B? Explain.
6. What does it mean if $\oint \vec{B} \cdot d\vec{\ell} \neq 0$ ? Explain.
7. A thin wire carries a uniform current $I$. This current produces a magnetic field, B. Up until now, you've always been told that magnetic fields loop around a current-carrying wire (Figure a. below) But how do you know that there are not other components to the magnetic field? Perhaps the magnetic field has a z-component (Figure b.) or a radial component (Figure c.)

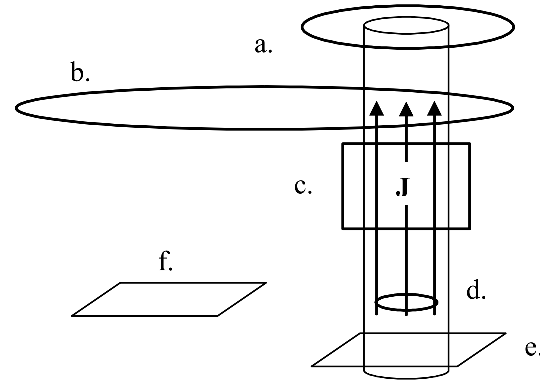
   Can you think of any convincing arguments for why there shouldn?t be a z- or s- component? It might be useful to consider symmetry, Maxwell's equations, and any laws that have recently been covered in class.

   (a) Give an argument for why there cannot be a $z$-component? (Figure b.)
   (b) Give an argument for why there cannot be a $s$-component? (Figure c.)



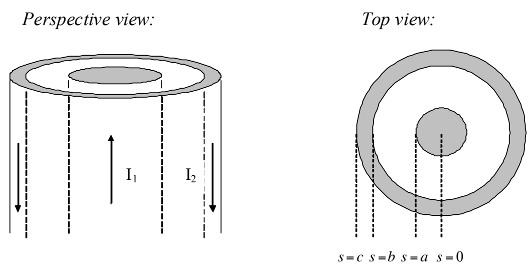
8. Consider the long fat cylindrical wire with a known, azimuthally symmetric current density J shown below. Look at the various loops shown in the figure, and decide what information, if any, Ampere's law applied to each loop might provide about B.

(a) Loop a: (It is centered on wire.)
(b) Loop b:
(c) Loop c:
(d) Loop d: (also centered)
(e) Loop e: (also centered)
(f) Loop f:



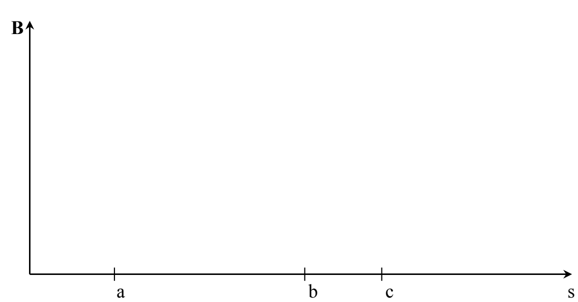

9. A coaxial cable consists of two conductors: an inner wire and an outer cylindrical shell. Both the inner wire and outer shell can carry currents.

The center wire has radius $a$, and a current density, $J$, that is is uniformly distributed through the cross-sectional area of the wire.




Consider the following questions qualitatively:

(a) Is there a magnetic field inside either of the two conductors?
(b) How should $\vec{I}_1$ and $\vec{I}_2$ compare in order to produce no magnetic field outside of the coax?
(c) For the case of $I_1 = I_2$, sketch a qualitative graph of B in the four regions: $s < a$, $a < s < b$, $b < s < c$, and $s > c$. Try to do this without solving the problem first! (e.g., is B zero, growing, falling?)



(d) Now calculate $\vec{B}$ in the four regions using Ampere's Law (assuming $I_1 = I_2$):

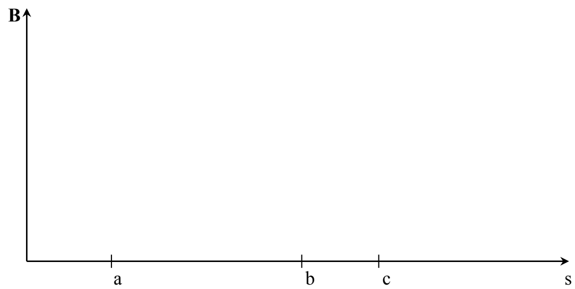
i. $s < a$

ii. $a < s < b$

iii. $b < s < c$

iv. $s > c$

(e) Based on your calculations, qualitatively graph $\vec{B}$ in the four regions again. How does it compare to your prediction?




(f) If $I$ is 1 Amp (not an unreasonable size):

i. What is the maximum value of $B$ produced? You may find the following number useful: $\mu_0 = 4\pi \times 10^{-7}$ N/A$^2$.

ii. Where is that maximum B produced?

iii. What is $B$ at a distance of 10cm from the wire?

iv. At either of these places, how does it compare with the earth's magnetic field (about $5 \times 10^{-5}$ T)?

10. **The Solenoid**

A (very long) solenoid consists of $n$ closely wound turns per unit length on a cylinder of radius $R$, each carrying a steady current $I$ (Fig 5.34). [The point of making the windings so close is that one can then pretend each turn is circular. If this troubles you (after all, there is a net current I in the direction of the solenoid?s axis, no matter how tight the winding), picture instead a sheet of aluminum foil wrapped around the cylinder, carrying the equivalent uniform surface current $K = nI$ (Fig. 5.35).

(a) First, what is the direction of $\vec{B}$? Consider each component:

i. Do you think there could be a radial component? Why or why not?

ii. How about an azimuthal component? ($\phi$ direction?)

iii. Along the axis of the solenoid?

(b) For your direction, what shape Amperian loop would work? What must the field be outside the solenoid? Why?

(c) Find $\vec{B}$ for this (long) solenoid.

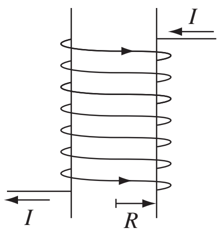



FIGURE 5.34

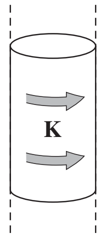



FIGURE 5.35

11. Griffiths 5.14b
12. Griffiths 5.16