Exam 1 Corrected, for you to make solutions
Physics 110, Oct 2, 2018

In addition to this page, I will include the "purple equation sheet" from Griffiths.

No phones or other device that connects to the internet.

You may use a calculator, though I don't think you'll need it.

Present clear and complete answers:

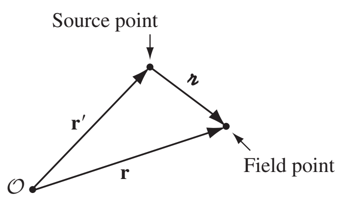
Explain your answers clearly but briefly. You want to aim for a level of solution that someone taking this class would be able to understand. A diagram and a few words may help.

Start calculations with first principles: things like definitions ($\vec{E} \equiv \frac{\vec{F}}{Q}$) or empirical laws (like Coulomb's Law or Newton's Laws) or conservation laws.

Check time:

The point values for each problem are shown next to the question number. Time yourself accordingly. The total value of the exam is 100 points. **Good luck!**

Some definitions:



Some more math:

$$\mathscr{r}^2 = r^2 + r'^2 - 2rr'\cos\alpha$$

$$V_s = \frac{4}{3}\pi R^3$$

Helpful Equations:

$$\vec{F} = \frac{qQ}{4\pi\epsilon_0 \mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}$$

$$\oint \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\epsilon_0}$$

$$V(r) = -\int_{ref}^{r} \vec{E}\cdot d\vec{\ell}$$

Helpful Integrals:

$$\int \sqrt{1-x^2}dx = \frac{1}{2}[x\sqrt{1-x^2} + \sin^{-1}x]$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x$$

$$\int \frac{xdx}{\sqrt{1-x^2}} = -\sqrt{1-x^2}$$

$$\int \frac{x^2dx}{\sqrt{1-x^2}} = -\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x$$

Helpful Taylor series expansions (for small ϵ):

$$e^{\epsilon} \approx 1 + \epsilon + \ldots$$

$$\ln(1+\epsilon) \approx \epsilon + \ldots$$

$$(1+\epsilon)^n \approx 1 + n\epsilon + \ldots$$

NAME:______________________________

1. (5 points) Calculate the gradient of this function, $f = 3x^2 + xz + y^3$

2. (5 points) Calculate the divergence of this vector function:

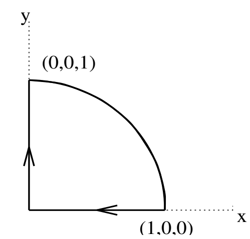
$$\vec{A} = \frac{\sin\theta}{r}\hat{r}$$

3. (10 points) Could this $\vec{E}$ be an electric field? Why or Why not?

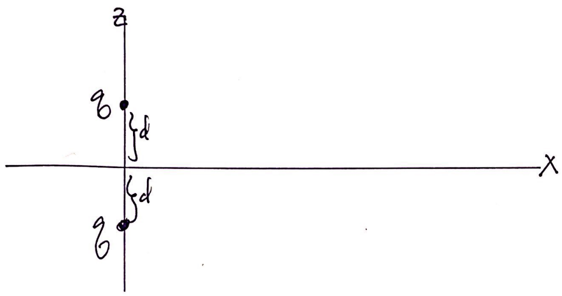
$$\vec{E} = x\hat{y}$$

4. (10 points) An electric field is given by $\vec{E} = \frac{c}{r^3}\hat{r}$. Find the potential at r, assuming a reference point at infinity.

5. (20 points) Verify Stokes' Theorem (the curl theorem) by considering $\vec{v} = y\hat{x} - x\hat{y}$ and the closed path shown in the figure. Hint: The equation of a circle of radius 1 is $x^2 + y^2 = 1$.



6. (15 points) Two equal positive point charges q are located on the z-axis as shown. One at $(0, 0, d)$, the other at $(0, 0, -d)$. Find the electric field at position x along the x-axis.

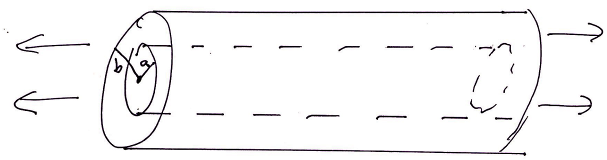


7. (20 points) An infinitely long cylinder with a hollow center (thicker than a shell) has a uniform volume charge distribution, ρ. The cylinder has inner radius a and outer radius b. Using s as radial to the cylinder:

   (a) What is the electric field outside the cylinder (for $s > b$) ?

   (b) What is the electric field in the solid part of the cylinder (for $a < s < b$) ?

   (c) What is the electric field in the hollow part, where $s < a$?



8. (15 points) A hemispherical shell of radius R has a surface charge distribution $\sigma = C\cos^2\theta$ where C is a positive constant. What is the potential at the middle of the base of the hemisphere?