9/11 In Class – More Electric Fields from Continuous Charge Distributions

The electric field

For one point charge, q, the electric field is

$$\vec{\boldsymbol{E}} = k\frac{q}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}$$

where $\vec{\mathscr{r}}$ is the vector from the source charge q to the point at which you want to find the field.

If you have n source charges, you add the electric fields (as vectors!) due to each point charge.

$$\vec{\boldsymbol{E}} = \vec{\boldsymbol{E}}_1 + \vec{\boldsymbol{E}}_2 + \cdots + \vec{\boldsymbol{E}}_n$$

For continuous charge distributions, you can pretend to add up (integrate) the electric fields due to infinitesimally small pieces of the charge–each of those acting like a source point charge.

$$d\vec{\boldsymbol{E}} = k\frac{dq}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}$$

and

$$\vec{\boldsymbol{E}} = \int d\vec{\boldsymbol{E}}$$

I left problems 1 and 2 even though most groups had finished. All groups were working on problem 3 last time. I have made comments in yellow post-it notes on your Jamboards. Please continue on the same Jamboards.

(I have added one new problem today.)

1. If you haven't already done this one, start with Griffiths 2.5 (copied here).

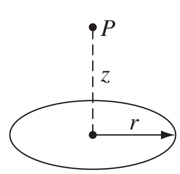
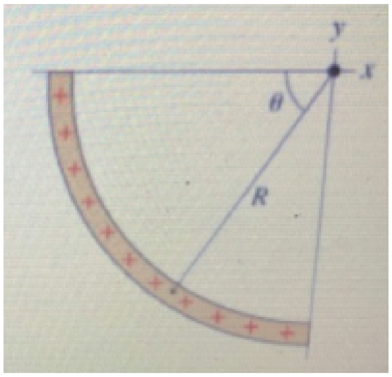


FIGURE 2.9

Problem 2.5 Find the electric field a distance z above the center of a circular loop of radius r (Fig. 2.9) that carries a uniform line charge λ.

(Save your answer!)

2. Instead of a full ring of charge, imagine only the 1/4 of the ring in the third quadrant has charge. The charge density is λ. The full ring would have radius R and would be centered at the origin. Find the electric field at the origin.



3. Using your answer to the first problem on this sheet (= Griffiths 2.5), find the electric field above the center of a disk of charge. Instead of the usual dq as a curvilinear square, think of dq now as a ring with thickness dr. Sketch this! This is Griffiths problem 2.6. Do the rest of his problem as well (shown here).

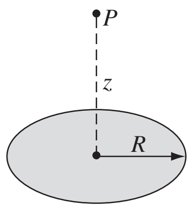


FIGURE 2.10

Problem 2.6 Find the electric field a distance z above the center of a flat circular disk of radius R (Fig. 2.10) that carries a uniform surface charge σ. What does your formula give in the limit $R \to \infty$? Also check the case $z \gg R$.

4. Gauss's Law: Recall that the (electric) flux Φ_E, is given by

$$\Phi_E = \int \vec{\boldsymbol{E}} \cdot d\vec{a}$$

Evaluate the total flux for a point charge. Use a sphere as your surface, and imagine the point charge at the center of the sphere.

If you substitute $k = \frac{1}{4\pi\epsilon_0}$ and let $q \to q_{enc}$, you might recognize Gauss's Law. (If you don't remember Gauss's Law, look it up to check your answer.)

5. Use Gauss's Law to find the electric field a distance r away from a(n infinitely) long line of charge. The line has uniform charge density λ. Hint: recall that you need a Gaussian Surface that is symmatric around the charge distribution (or else Gauss's Law is not helpful and you have to integrate the hard way.)

6. In class we found the electric field a distance h above the center of a line of charge of length d to be

$$\vec{\boldsymbol{E}} = \frac{k\lambda d}{h\sqrt{h^2 + (d/2)^2}}\hat{\boldsymbol{z}}$$

   Take the limit of this, and see if you get the same answer has the previous problem. (What limit makes it like the previous problem?)

New Problem here

7. Find the divergences of $r^2\hat{r}$ and $\hat{r}/r^2$. Please do Griffiths 1.39, copied here:

   Problem 1.39

   (a) Check the divergence theorem for the function $\mathbf{v}_1 = r^2\hat{\mathbf{r}}$, using as your volume the sphere of radius R, centered at the origin.

   (b) Do the same for $\mathbf{v}_2 = (1/r^2)\hat{\mathbf{r}}$. (If the answer surprises you, look back at Prob. 1.16.)