Phys 125
Boyda

Early-quantum redux. Over the break, you solve as many problems below as possible, and I will use the credit to fill up existing homework scores. Supposing you gain 8 points of credit on this assignment, I could use 3 points to turn a 9/12 into a 12/12 and the remaining 5 points to turn a 6/12 into an 11/12. If a solution to a problem is already posted online, you get credit only for detailed step-by-step solutions. Problems are due at the beginning of Tuesday class (Oct. 24).

Problems

1. Using Euler's formula, prove the following double and half angle formulas:

$$\begin{aligned}\sin 2\alpha &= 2\sin\alpha\cos\alpha \\ \cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\ \sin^2\frac{\alpha}{2} &= \frac{1-\cos\alpha}{2} \\ \cos^2\frac{\alpha}{2} &= \frac{1+\cos\alpha}{2}\end{aligned}$$

2. For complex number $z = Re(z) + iIm(z)$, what is $|z|^2$ in terms of $Re(z)$ and $Im(z)$?

3. Solve the differential equation for a damped harmonic oscillator

$$m\ddot{x}(t) + b\dot{x}(t) + kx(t) = 0$$

by *complexifying* x. That is, find the general complex $\xi(t)$ that solves the differential equation and then take $x = Re(\xi)$. Let's say this is an underdamped oscillator ($b^2 < 4km$) with $x(0) = 0$ and $\dot{x}(0) = v_0$. Sketch a graph for $x(t)$.

4. Normalize the spin-1/2 state

$$|\psi\rangle \to \begin{pmatrix} 2 \\ 3i \end{pmatrix},$$

given in the basis of S_z eigenstates. Before computing anything, which of $\langle S_x\rangle$, $\langle S_y\rangle$, $\langle S_z\rangle$ do you expect to be largest in magnitude, and why? Check your guess.

5. Write the spin-1/2 operators $\hat{S}_x$, $\hat{S}_y$, $\hat{S}_z$ in the basis of S_x eigenstates. (Note that we almost always write them in the basis of S_z eigenstates.) By matrix multiplication, show that the commutation relation $[\hat{S}_x, \hat{S}_y] = i\hbar\hat{S}_z$ is satisfied.

6. Find eigenvalues and normalized eigenvectors for the matrix

$$H = \begin{pmatrix} 5 & -\sqrt{2} \\ -\sqrt{2} & 4 \end{pmatrix}.$$

7. Referring to the previous problem, compute

$$e^{-iHt} \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

8. 1.13, 1.14, 3.5, 4.8, and any previously assigned problem from chapter 4 that you missed.