Phys 125
Boyda

Reading as noted below. The problems are due on Friday, November 10, at 4 p.m. in my box.

Reading

- For Tuesday 11/7: Townsend §7.1 − 7.5.
- For Thursday 11/9: Townsend §7.8.

Problems

1. A particle of mass m is in a potential well

$$V(x) = \begin{cases} 0 & 0 \le x \le 1 \\ V_0 & \text{otherwise} \end{cases},$$

with position x measured in angstroms.

   (a) Complete the derivation from class to find the bound state energies and the corresponding wavefunctions $\psi(x)$ when $2mV_0/\hbar^2 = 16$, keeping at least three significant digits for parameters k and q. You should find two distinct bound states. You may leave an overall normalization constant in your solution.

   (b) Plot $|\psi(x)|^2$ for $x \in [-1, 2]$ in MATLAB or a program of your choice. To plot a piecewise-defined function in MATLAB, you can simply define multiple x variables, say $x1$, $x2$, $x3$, define functions on each of those, $f1(x1)$, $f2(x2)$, $f3(x3)$, and then call

$$>> \text{plot}(x1, f1(x1), x2, f2(x2), x3, f3(x3)).$$

   Pick any convenient value for the normalization constant to execute your plots.

2. A free electron with 10 eV kinetic energy and corresponding $\langle p_x \rangle = p_0$ has a Gaussian momentum-space wavefunction with spread comparable to the momentum, $\Delta p_x = p_0$. That is,

$$\psi(p) = \sqrt{\frac{a}{\hbar\sqrt{\pi}}} e^{-(p-p_0)^2 a^2/2\hbar^2},$$

with $a = \hbar/\sqrt{2}\Delta p_x = \hbar/\sqrt{2}p_0$ (cf. Townsend eqn. 6.68, 6.72).

(a) Compute $|\psi(x,t)|^2$ for later times t. To do so, use a u-substitution in the dp integral to show that the wavefunction is the same as the one derived in problem 6.4 with the substitution $x \to x - p_0 t/m$, up to a time- and position-dependent overall phase.

(b) (Revised.) Plot $|\psi(x,t)|^2$ on the computer in six intervals of time from $t = 0$ to $t = .2 \cdot 10^{-15}$ s. A nice way to do this in MATLAB is to create a fixed range

$$>> x = -1\text{e-}7 : 1\text{e-}7/1000 : 1\text{e-}7;$$

and then define a function $rho(t) = |\psi(x,t)|^2$ evaluated on the array x. You can plot all six time steps with the commands

```
>> times = (0:4:20)*1e-17;
>> for t = times;
>>       plot(x,rho(t)); hold on;
>> end
```

What behavior do you observe in the time evolution of the free electron?

3. Townsend problems 6.25, 7.5, 7.7.