Physics 140,Spring 2016

In Class 9/16 – 4-vectors

1. Write out the 4-vector x' from the Lorentz transformation as follows:

$$x' = \Lambda x = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

   Show that if you plug in for β and x^0, you get the Lorentz Transformations as written in Eq 3.1 in the textbook..

2. Write out the 4-vector x' by calculating this matrix multiplication:

$$x' = \Lambda x = \begin{pmatrix} \Lambda^0_0 & \Lambda^0_1 & \Lambda^0_2 & \Lambda^0_3 \\ \Lambda^1_0 & \Lambda^1_1 & \Lambda^1_2 & \Lambda^1_3 \\ \Lambda^2_0 & \Lambda^2_1 & \Lambda^2_2 & \Lambda^2_3 \\ \Lambda^3_0 & \Lambda^3_1 & \Lambda^3_2 & \Lambda^3_3 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

   Check that this gives what you would expect from $x^{\mu\prime} = \Sigma \Lambda^\mu_\nu x^\nu$ for all values of μ.

3. We define

$$x^\mu \equiv \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

   This is called a contravariant four-vector (it's sort of the ordinary one you are used to.)

   (In my mind, this is very dangerous notation. It is only by context you will know whether x^μ means the 4-vector or a single component. It is true, that's usually clear, but you'll see right below why I use it (along with Griffiths.))

   We also define a covariant version:

$$x_\mu = (x^0, -x^1, -x^2, -x^3)$$

   This gives a nice way to write a 4-vector dot product:

$$a \cdot b = a_\mu b^\mu = a^0 b^0 - \vec{a} \cdot \vec{b} = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$$

   Write $(x)^2 = x \cdot x$ where x is the position-time four vector.

4. Calculate $a \cdot b$ given:

$$a = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \qquad b = \begin{pmatrix} 8 \\ 7 \\ -6 \\ 5 \end{pmatrix}$$

5. Invariants are quantities that are the same in all inertial reference frames. $(x)^2$ is the first one. Show that $(x')^2 = (x)^2$