Physics 140 Fall 2016

9/21 In Class Problems 4– Energy and Momentum

If you didn't finish the last problem from last time, please do that now.

We have seen that relativistic velocity, $\vec{\eta}$ is equal to $\gamma\vec{v}$. Classically, momentum is $m\vec{v}$. So the question becomes: for relativistic momentum, should we use $m\vec{v}$ or $m\vec{\eta}$? We choose $\vec{p} = \gamma m\vec{v}$.

1. Why choose to define relativistic $\vec{p} = \gamma m\vec{v}$?
   Imagine a scenario where in a particle of mass $5m$ at rest in lab frame decays into two particles of mass $2m$ each. (I chose numbers so the velocities work out beautifully.) I solved this for you, and the speed of each decay particle is $\frac{3}{5}c$. One of my favorite numbers :) In the lab frame, conservation of momentum is easy, it's zero before collision, and the total must be zero after. Rotate so the decay particles (the $2m$'s) go off such that one goes in the positive x direction, one in the negative x direction.

   (a) Use the classical definition of momentum $p = mv$ and check conservation of momentum in the rest frame of one of the decay particles. So choose frame S' to move at speed $\frac{3}{5}c$ relative to the lab. (This will be the rest frame of one of the decay particles, so one of the velocity additions is easy. Use the classical definition of relativity, but do relativistic velocity addition. The whole point is why we can't use the old definition. Or at least, why it would be a big problem.) Is classical momentum conserved from the standpoint of an observer in frame S'? Comment.

   (b) Now repeat that analysis using the relativistic momentum of the particles in frame S'. (Still zero in the lab frame.) Is relativistic momentum conserved?
   Thus the choice of the definition of relativistic momentum.

   (c) Is either kind of momentum (as a 3-vector) invariant?

2. The relativistic momentum 4-vector is defined as $p^\mu = (\frac{E}{c}, p_x, p_y, p_z)$ where E is the relativistic energy, $E = \gamma mc^2$ and $\vec{p} = \gamma m\vec{v}$.

   (a) Show that $p_\mu p^\mu$ is invariant. Do you recognize this equation from Physics 60?

   (b) Write down the momentum 4-vector for one of the decay particles in the lab frame (from Problem 1).

   (c) Use conservation of relativistic energy in the lab frame to find the speed of the decay particles (in the lab frame) It should obviously be $\frac{3}{5}c$.

   (d) Now check conservation of relativistic energy in the frame where one of the decay particles is at rest. Is relativistic energy conserved in S and S'? Is relativistic energy invariant?